Prove $\lim_{x\to 5}(x^2-3x) = 10$

Choose $\delta = \min(3, \frac{\varepsilon}{10})$

(11) Last Step: Better describe Choice of δ

(1) Start with the "if part of the epsilon-delta def.

$0 < |x-5| < \delta$

Case 1: If $\frac{\varepsilon}{10} < 3$, Choose $\delta = \frac{\varepsilon}{10}$

$\Rightarrow |x-5| < \frac{\varepsilon}{10}$

so now we can add this part.

$\Rightarrow |x-5| < \frac{\varepsilon}{|x+2|}$

(3) Work backwards to isolate a $|x-5|$ seen at the start (in step 1). [Factoring helps here.]

$\Rightarrow |x-5||x+2| < \varepsilon$

$\Rightarrow |(x^2-3x)-10| < \varepsilon$

(2) put as your last step the "then" part of the epsilon-delta def.

(4) We are tempted to use $\delta = \frac{\varepsilon}{|x+2|}$, but can't as δ should only depend on ε.

(5) Instead, pick some individual δ. Be sure to pick one small enough avoid domain issues. For quadratic functions, this isn't a concern. Some values will lead to general δ-values that are "nicer" than others (e.g. $\delta = \varepsilon$ is pretty, $\delta = \frac{\varepsilon}{7}$ is less so) but all should work for quadratic functions.

Note

(10) Of course there's another case — what if $3 \le \frac{\varepsilon}{10}$?

But then, choosing $\delta = 3$ lets us link things up too...

so now we add this part

Case 2: If $3 \le \frac{\varepsilon}{10}$ Choose $\delta = 3$

$0 < |x-5| < \delta$ (same starting statement)

$\Rightarrow |x-5| < 3$ (1) key step as in this case $3 \le \frac{\varepsilon}{10}$

$\Rightarrow |x-5| < \frac{\varepsilon}{10}$

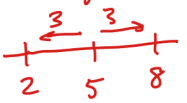
$\Rightarrow |x-5| < \frac{\varepsilon}{|x+2|}$

$\Rightarrow |x-5||x+2| < \varepsilon$

$\Rightarrow |(x^2-3x)-10| < \varepsilon$

same as before

(6) Suppose we pick $\delta = 3$. This gives us a delta-band of $(2,8)$



(7) Now think about how small $\frac{\varepsilon}{|x+2|}$ gets on this interval.

At $x = 8$, we would have $\frac{\varepsilon}{10}$

So if x is in $(2,8)$ then $\frac{\varepsilon}{10} < \frac{\varepsilon}{|x+2|}$

Another way to write this is $|x-5| < 3$

(8) So we could say for any $\delta < 3$...

If $|x-5| < \delta$

(then as $\delta < 3$) $\Rightarrow |x-5| < 3$

$\Rightarrow \frac{\varepsilon}{10} < \frac{\varepsilon}{|x+2|}$ by using the result of step 7 above

(9) which means if we take $\delta = \frac{\varepsilon}{10}$ AND $\delta < 3$ we can link things up above