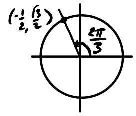
2a

$$\operatorname{arcsec}^2\left(\csc\frac{2\pi}{3}\right) = \Big(\underbrace{\operatorname{arcsec}\left(\tfrac{2}{\sqrt3}\right)}_{\text{call this }\theta}\Big)^2$$



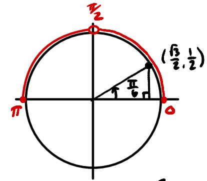
$\sin\frac{2\pi}{3} = \frac{\sqrt3}{2}$

$\csc\frac{2\pi}{3} = \frac{2}{\sqrt3}$

$\sec\theta = \frac{2}{\sqrt3}$

$\cos\theta = \frac{\sqrt3}{2}$

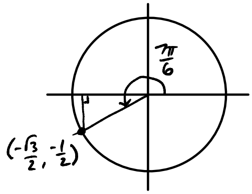
Recall the range of arcsec is $[0,\pi]$ (except $\frac{\pi}{2}$)



So we seek $\theta^2 = \left(\frac{\pi}{6}\right)^2 = \boxed{\frac{\pi^2}{36}}$

2b

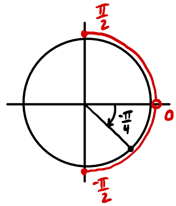
$$\cos^2\frac{7\pi}{6} = \left(\cos\frac{7\pi}{6}\right)^2 = \left(-\frac{\sqrt3}{2}\right)^2 = \boxed{\frac{3}{4}}$$



2c

$$\underbrace{\operatorname{arccsc}(-\sqrt2)}_{\text{call this }\theta} \to \csc\theta = -\sqrt2$$

$$\sin\theta = \frac{1}{-\sqrt2} = \frac{-1}{\sqrt2}\cdot\frac{\sqrt2}{\sqrt2} = \frac{-\sqrt2}{2}$$

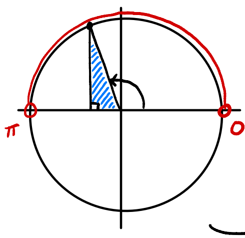


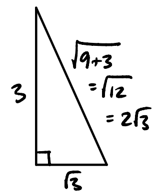
Recall the range of arccsc is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (except 0)

So, $\boxed{\theta = -\frac{\pi}{4}}$

2d

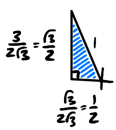
$$5\underbrace{\operatorname{arccot}\left(\tfrac{-\sqrt3}{3}\right)}_{\text{call this }\theta}$$





scale up to see the ratio $-\frac{\sqrt3}{3}$ more easily and then use Pythag. thm to find 3rd side

Now scale down to find $\cos\theta$ and $\sin\theta$



$\cos\theta = -\frac{1}{2}$

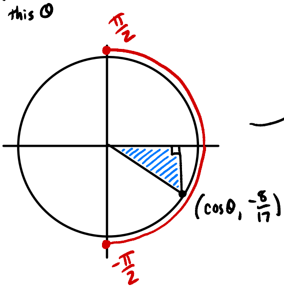
$\sin\theta = \frac{\sqrt3}{2}$

Recall the range of arccot is $(0,\pi)$

$\boxed{\theta = \frac{2\pi}{3}}$

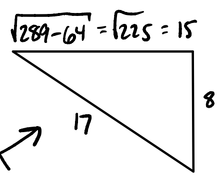
2e

$$\cot\Big(\underbrace{\arcsin\left(\tfrac{-8}{17}\right)}_{\text{call this }\theta}\Big)$$

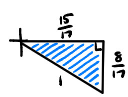


scale up to avoid fractions and use Pythag to find 3rd side

$\sqrt{289-64} = \sqrt{225} = 15$

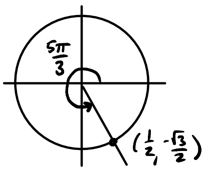


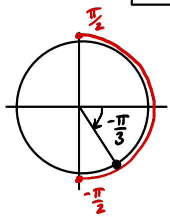
then scale down to find $\cos\theta$ and $\sin\theta$



$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{15}{17}}{\frac{-8}{17}} = \boxed{\frac{-15}{8}}$$

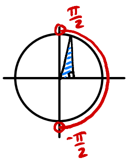
(2f) $\arcsin\left(\sin \frac{5\pi}{3}\right) = \arcsin\left(-\frac{\sqrt{3}}{2}\right) = \boxed{-\frac{\pi}{3}}$



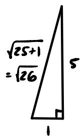


Recall the range of arcsin is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

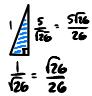
(2g) $\cos(\underbrace{\arctan 5}_{\text{call this } \theta})$



Scale up so ratio of 5 (i.e. $\frac{5}{1}$) is easier to see and use Pythag. then to find 3^{rd} side

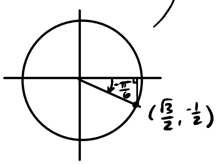


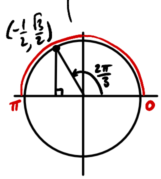
then scale down to find $\cos\theta$ and $\sin\theta$



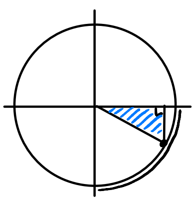
$\cos\theta = \boxed{\frac{\sqrt{26}}{26}}$

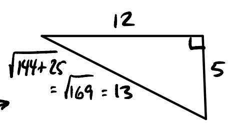
(2h) $\arccos\left(\sin\left(-\frac{\pi}{6}\right)\right) = \arccos\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$



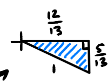


(2i) Find $\sin t$ if $\frac{3\pi}{2} < t < 2\pi$ and $\cot t = -\frac{12}{5}$





Scale up so that ratio of $-\frac{12}{5}$ is easier to see, then solve for the 3^{rd} side with Pythag. theorem.

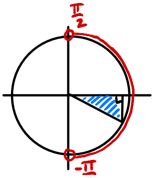


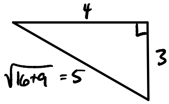
now scale down to find $\cos\theta$ and $\sin\theta$

$\cos\theta = \frac{12}{13}$

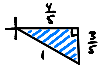
$\sin\theta = \boxed{\frac{-5}{13}}$

2j $\underbrace{\cos(\arctan(-\tfrac{3}{4}))}_{\text{call this } \theta}$





Scale up to make ratio $\frac{-3}{4}$ easier to see, and then find third side with Pythag. thm

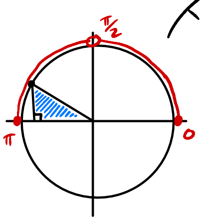


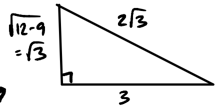
Then scale down to find $\cos\theta$ and $\sin\theta$

$\sin\theta = \frac{-3}{5}$

$\cos\theta = \boxed{\frac{4}{5}}$

2k $\underbrace{\text{arcsec}\left(\frac{-2\sqrt{3}}{3}\right)}_{\text{call this } \theta}$

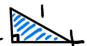




Scale up to make ratio of $\frac{-2\sqrt{3}}{3}$ easier to see. Then use Pythag. thm to find third side.

then scale down to find $\cos\theta$ and $\sin\theta$

$\frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$



$\frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

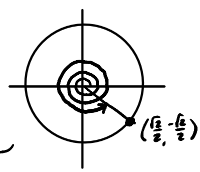
$\underbrace{\cos\theta = -\frac{\sqrt{3}}{2} \quad \sin\theta = \frac{1}{2}}_{\text{we recognize this angle!}}$

Recall the range of arcsec is $[0,\pi]$ (except $\frac{\pi}{2}$)

$\theta = \boxed{\frac{5\pi}{6}}$

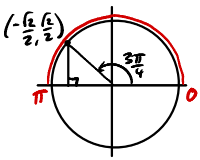
2l $\arccos\left(\sin\frac{23\pi}{4}\right)$

$= \arccos\left(\sin\left(\left(5\tfrac{3}{4}\right)\pi\right)\right) \rightarrow$



$= \arccos\left(-\frac{\sqrt{2}}{2}\right)$

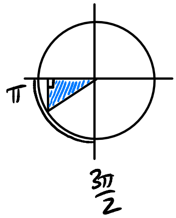
$= \boxed{\frac{3\pi}{4}}$



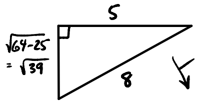
Recall the range of arccos is $[0,\pi]$

2m

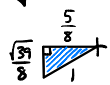
Find $\cot t$ if $\pi < t < \frac{3\pi}{2}$ and $\sec t = \frac{-8}{5}$



scale up so ratio of $\frac{-8}{5}$ is easier to see, and then use pythag. thm to find 3rd side



Then scale down to find $\cos t$ and $\sin t$



$$\cot t = \frac{\cos t}{\sin t} = \frac{-\frac{5}{8}}{-\frac{\sqrt{39}}{8}} = \frac{5}{\sqrt{39}} \cdot \frac{\sqrt{39}}{\sqrt{39}} = \boxed{\frac{-5\sqrt{39}}{39}}$$