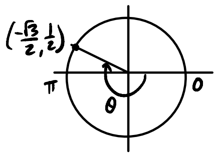
(1a) $\tan\left(-\frac{7\pi}{6}\right) = \tan\left(-1\tfrac{1}{6}\cdot\pi\right)$

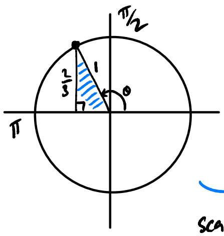


$$= \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{-\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{3}}$$

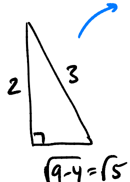
since $\sin\theta = \frac{1}{2}$, $\cos\theta = -\frac{\sqrt{3}}{2}$

and $\tan\theta = \frac{\sin\theta}{\cos\theta}$

(1b) $\sin\theta = \frac{2}{3}$ $\quad \frac{\pi}{2} < \theta < \pi$



scale up to avoid fractions



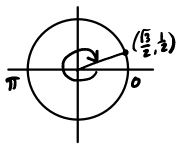
$\rightarrow \cos\theta = \frac{-\sqrt{5}}{3}$

So, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$$= \frac{-\frac{\sqrt{5}}{3}}{\frac{2}{3}} = \boxed{\frac{-\sqrt{5}}{2}}$$

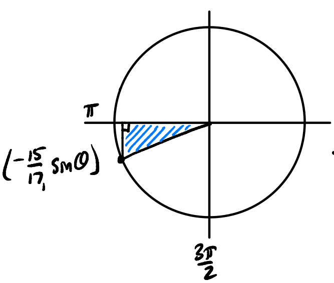
(1c) $\cos\left(\frac{-11\pi}{6}\right) = \cos\left(-2\pi + \frac{\pi}{6}\right)$

$= \cos\left(\frac{\pi}{6}\right) = \boxed{\frac{\sqrt{3}}{2}}$

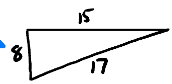


(1d) $\cos t = \frac{-15}{17}$

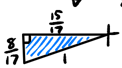
$\sqrt{17^2 - 15^2} = \sqrt{64} = 8$



scale up to avoid fractions



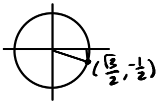
Scale back down to get $\sin\theta$



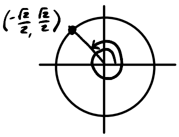
$\sin\theta = \frac{-8}{17}$

$\csc\theta = \boxed{\frac{-17}{8}}$

(1e) $\tan\left(-\frac{\pi}{6}\right) = \dfrac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{3}}$

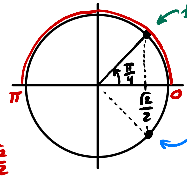


(1f) $\arccos\left(\sin \frac{11\pi}{4}\right) = \arccos\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\pi}{4}}$



Recall the range of arccos is $[0, \pi]$

So we seek θ in this range where $\cos\theta = \frac{\sqrt{2}}{2}$



this angle gives us the cosine value sought and is in the range required by arccos.

note this angle gives us the same cosine, but it is not in the range required by arccos

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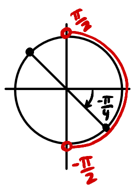
(1g) $\cos(\underbrace{\arctan(-1)}_{\theta}) \rightarrow$



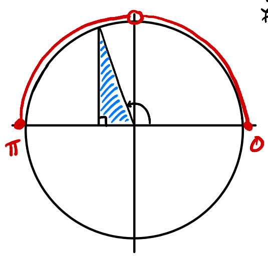
recall range of arctan is $-\frac{\pi}{2}$ to $\frac{\pi}{2}$

So we want the bottom one

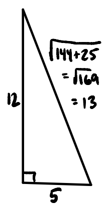
$= \cos\left(-\frac{\pi}{4}\right)$

$= \boxed{\frac{\sqrt{2}}{2}}$

(1h) $\sec(\underbrace{\operatorname{arccot}\left(-\frac{5}{12}\right)}_{\text{Call this } \theta})$



Scale up to avoid fractions



Scale down to get $\cos\theta$ and $\sin\theta$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{5}{13}} = \boxed{-\frac{13}{5}}$

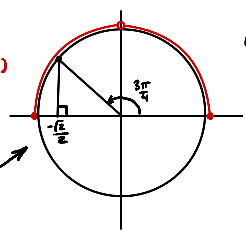
(1i) $\underbrace{\operatorname{arcsec}(-\sqrt{2})}_{\text{call this } \theta}$

recall the range of arcsec is $[0, \pi]$ (excluding $\frac{\pi}{2}$)

$\theta = \boxed{\frac{3\pi}{4}}$

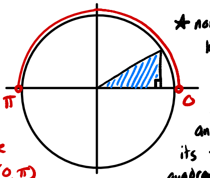
$\sec\theta = -\sqrt{2}$

$\cos\theta = \frac{1}{-\sqrt{2}} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$



(1j) $\sec(\underbrace{\operatorname{arccot}\frac{x}{3}}_{\text{call this } \theta})$

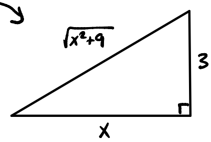
$\cot\theta = \frac{x}{3}$



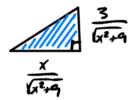
Recall the range of arccot is $(0,\pi)$

★ note, without knowing whether $\cot\theta$ is positive or negative, we can't decide whether angle θ should have its terminal side in quadrant I or II. We have drawn it here in quadrant I.

When we scale the triangle up to avoid fractions, the "sign information" is lost anyways, though



then we Scale back down to get $\sin\theta$ and $\cos\theta$



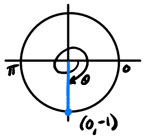
Note: $\sin\theta$ is clearly positive while the sign of $\cos\theta$ is the same as the sign of x

$$\sec\theta = \frac{1}{\frac{x}{\sqrt{x^2+9}}} = \frac{\sqrt{x^2+9}}{x}$$

(1k) $\arctan\left(\sin\left(-\frac{5\pi}{2}\right)\right)$

First, note

$\sin\left(-\frac{5\pi}{2}\right) = -1$ as

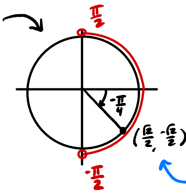


Then, $\underbrace{\arctan(-1)}_{\text{call this } \theta}$

$\tan\theta = -1$

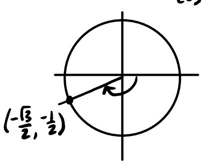
Recall, the range of arctan is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

So the value we seek is $\boxed{\frac{-\pi}{4}}$



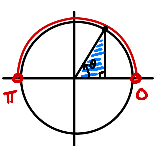
note $\tan\theta = \frac{\sin\theta}{\cos\theta}$.
So $\tan\theta = -1$ implies $\sin\theta$ and $\cos\theta$ agree in magnitude but are opposite in sign.

(1l) $$\sec\left(-\frac{5\pi}{6}\right) = \frac{1}{\cos\left(-\frac{5\pi}{6}\right)} = \frac{1}{-\frac{\sqrt{3}}{2}} = \frac{-2}{\sqrt{3}} = \frac{-2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$$



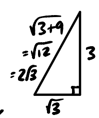
(1m) $$9\operatorname{arccot}^2\frac{\sqrt{3}}{3} = 9\left(\underbrace{\operatorname{arccot}\frac{\sqrt{3}}{3}}_{\text{call this } \theta}\right)^2$$

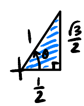
$\cot\theta = \frac{\sqrt{3}}{3} \rightarrow \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{3}}{3}$



Recall the range of arccot is $(0,\pi)$

Scale up so this ratio is easier to express, and then find third side by Pythag. Then





then scale down (divide by $2\sqrt{3}$ and rationalize) to find $\cos\theta$ and $\sin\theta$

note: the values here $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ are recognizable!

... and associated with $\theta = \frac{\pi}{3}$

$$9\operatorname{arccot}^2\frac{\sqrt{3}}{3}$$

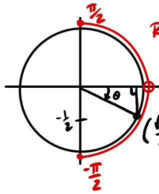
$$= 9\left(\frac{\pi}{3}\right)^2$$

$$= \frac{9\pi^2}{9}$$

$$= \boxed{\pi^2}$$

1n

$\underbrace{\text{arccsc}(-2)}_{\text{call this } \theta} \longrightarrow \csc\theta = -2 \longrightarrow \sin\theta = -\frac{1}{2}$



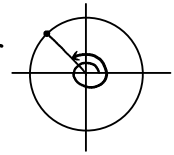
Recall the range of arccsc is $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (except 0)

$\longrightarrow \theta = \boxed{-\frac{\pi}{6}}$

1p

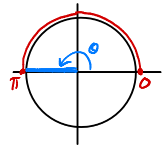
$\text{arccos}\left(\cot \frac{11\pi}{4}\right)$

$= \text{arccos}\left(\cot\left(\left(2\tfrac{3}{4}\right)\pi\right)\right)$



$= \underbrace{\text{arccos}(-1)}_{\text{call this } \theta}$

Recall arccos θ is always in $[0, \pi]$, so

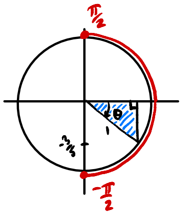


note $\cos(\pi) = -1$

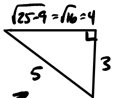
So $\text{arccos}\,\theta = \boxed{\pi}$

1o

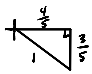
$\tan(\underbrace{\arcsin(-\tfrac{3}{5})}_{\text{call this } \theta}) \longrightarrow \sin\theta = \frac{-3}{5}$



scale up to avoid fractions



scale down to find $\cos\theta$ and $\sin\theta$

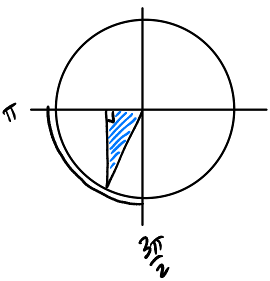


$\tan\theta = \frac{\sin\theta}{\cos\theta}$

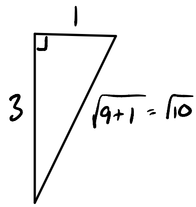
$= \frac{-\frac{3}{5}}{\frac{4}{5}} = \boxed{\frac{-3}{4}}$

1q

$\pi < t < \frac{3\pi}{2} \qquad \tan t = 3$

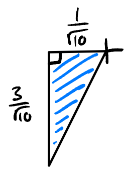


scale up so this ratio $(3 = \frac{3}{1})$ is easier to see, then use Pythag. thm to find 3rd side.



now scale back down to find $\cos\theta$ and $\sin\theta$

(note, we save any rationalizing denominators for the end – it's possible we won't need to do this depending on what we seek!)



$\longrightarrow \csc\theta = \frac{1}{\sin\theta}$

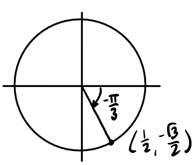
$= \frac{1}{\frac{-3}{\sqrt{10}}}$

$= \boxed{\frac{-\sqrt{10}}{3}}$

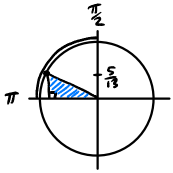
1r

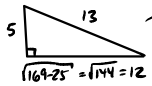
$\text{arccsc}\left(\cos\frac{-\pi}{3}\right) = \text{arccsc}\left(\frac{1}{2}\right) \rightarrow \boxed{\text{no value!}}$

recall $|\csc\theta| \geq 1$ always!



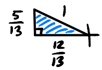
(1s) $\frac{\pi}{2} < \theta < \pi \qquad \sin\theta = \frac{5}{13}$





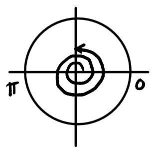
scale up to avoid fractions and use Pythag. Thm to find 3rd side

then scale down to find $\cos\theta$ and $\sin\theta$

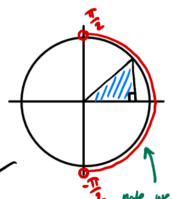


$$\rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{5}{13}}{-\frac{12}{13}} = \boxed{-\frac{5}{12}}$$

(1t) $\tan\frac{9\pi}{2} = \tan\left(4\tfrac{1}{2}\cdot\pi\right) = \frac{1}{0}$ (impossible!) $\boxed{\text{no value}}$



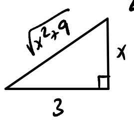
(1u) $\cos(\underbrace{\arctan\tfrac{x}{3}}_{\text{call this }\theta})$



Recall the range of arctan is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

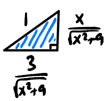
Further, notice on this range, cosine is always positive

note, we actually don't know in which quadrant θ lies (possibly I or IV) as it depends on the sign of x.



Scale up to make the ratio $\frac{x}{3}$ easier to see, and then use Pythag. thm to find 3rd side

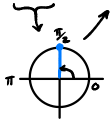
now scale down to find $\cos\theta$ and $\sin\theta$

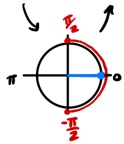


$$\rightarrow \boxed{\cos\theta = \frac{3}{\sqrt{x^2+9}}} = \frac{3}{\sqrt{x^2+9}}\cdot\frac{\sqrt{x^2+9}}{\sqrt{x^2+9}} = \boxed{\frac{3\sqrt{x^2+9}}{x^2+9}}$$

..or with the denominator rationalized

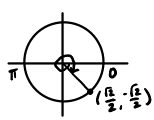
(1v) $\arcsin\left(\cos\frac{\pi}{2}\right) = \arcsin(0) = \boxed{0}$



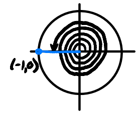


(1x) $\csc\left(-\frac{9\pi}{4}\right) = \csc\left(-2\tfrac{1}{4}\cdot\pi\right)$

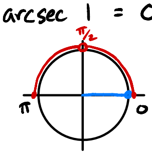
$$= \frac{1}{-\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}} = \boxed{-\sqrt{2}}$$



(1z) $\tan\left(\frac{54\pi}{6}\right) = \tan(9\pi) = \frac{0}{-1} = \boxed{0}$

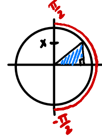


(1w) $\operatorname{arcsec} 1 = 0$

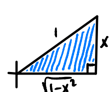


Recall, the range of arcsec is $[0, \pi]$ (except $\frac{\pi}{2}$)

(1y) $\cos(\underbrace{\arcsin x}_{\text{call this }\theta})$



Recall the range of arcsin is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and cosine is positive throughout this range



So $\cos\theta = \boxed{\sqrt{1-x^2}}$