4a)

$$\frac{1}{\csc\theta+\cot\theta}+\frac{\sec\theta+1}{\tan\theta}\stackrel{?}{=}2\csc\theta$$

$$=\frac{1}{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}}+\frac{\frac{1}{\cos\theta}+1}{\frac{\sin\theta}{\cos\theta}}$$

$$=\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$$

$$=\frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}+\frac{1+\cos\theta}{\sin\theta}$$

$$=\frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}+\frac{1+\cos\theta}{\sin\theta}$$

$$=\frac{\cancel{\sin\theta}(1\cancel{-\cos\theta})}{\sin^{\cancel{2}}\theta}+\frac{1\cancel{+\cos\theta}}{\sin\theta}$$

$$=\frac{2}{\sin\theta}\ \checkmark$$

$$=\frac{2}{\sin\theta}$$

4b)

$$\frac{1+\tan\theta}{1-\tan\theta}+\frac{1+\cot\theta}{1-\cot\theta}\stackrel{?}{=}0$$

$$=\frac{1+\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin\theta}{\cos\theta}}+\frac{1+\frac{\cos\theta}{\sin\theta}}{1-\frac{\cos\theta}{\sin\theta}}$$

$$=\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$$

$$=\frac{\cancel{\cos\theta}+\cancel{\sin\theta}}{\cos\theta-\sin\theta}-\frac{(\cancel{\sin\theta}+\cancel{\cos\theta})}{\cos\theta-\sin\theta}$$

$$=0\ \checkmark$$

4c)

$$\frac{1}{1+\cos\theta}-\frac{1}{1-\cos\theta}\stackrel{?}{=}\frac{2}{\sec\theta-\cos\theta}$$

$$=\frac{(1-\cos\theta)-(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$=\frac{-2\cos\theta}{1-\cos^2\theta}$$

$$=\frac{2}{\frac{1}{\cos\theta}-\cos\theta}$$

$$=\frac{2\cos\theta}{1-\cos^2\theta}$$

X Not an identity!

any θ with $\cos\theta\neq 0$ will be a counterexample! For example, $\theta=\frac{\pi}{6}$

4d)

$$(1+\sec x)(1-\cos x)\stackrel{?}{=}\tan x\cdot\sin x$$

$$=\left(1+\frac{1}{\cos x}\right)(1-\cos x)$$

$$=\cancel{1}+\frac{1}{\cos x}-\cos x-\cancel{1}$$

$$=\frac{1-\cos^2x}{\cos x}$$

$$=\frac{\sin^2x}{\cos x}\ \checkmark$$

$$=\frac{\sin x}{\cos x}\cdot\sin x$$

$$=\frac{\sin^2x}{\cos x}$$

4e)

$$\sec 2\theta\stackrel{?}{=}\frac{\sec^2\theta}{2-\sec^2\theta}$$

$$=\frac{1}{\cos 2\theta}$$

$$=\frac{1}{\cos^2\theta-\sin^2\theta}$$

$$=\frac{1}{\cos^2\theta-(1-\cos^2\theta)}$$

$$=\frac{1}{2\cos^2\theta-1}$$

$$=\frac{\frac{1}{\cos^2\theta}}{2-\frac{1}{\cos^2\theta}}$$

$$=\frac{1}{2\cos^2\theta-1}$$

✓

4f)

$$\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}\stackrel{?}{=}2\sec\theta$$

$$=\frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$=\frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$=\frac{1+2\sin\theta+1}{\cos\theta(1+\sin\theta)}$$

$$=\frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$=\frac{2(\cancel{1+\sin\theta})}{\cos\theta(\cancel{1+\sin\theta})}$$

$$=2\sec\theta\ \checkmark$$

4g $\dfrac{(\tan^2 x)\cdot(\csc^2 x)-1}{(\csc x)(\tan^2 x)(\sin x)} \stackrel{?}{=} 1$

$$= \frac{\frac{\cancel{\sin^2 x}}{\cos^2 x}\cdot\frac{1}{\cancel{\sin^2 x}}-1}{\frac{1}{\cancel{\sin x}}\cdot\frac{\sin^{\cancel{2}} x}{\cos^2 x}\cdot\sin x}$$

$$= \frac{\frac{1}{\cos^2 x}-1}{\frac{\sin^2 x}{\cos^2 x}}$$

mult top/bottom by $\cos^2 x$

$$= \frac{1-\cos^2 x}{\sin^2 x}$$

$$= \frac{\sin^2 x}{\sin^2 x}$$

$$= 1 \quad ✓$$

4h $\dfrac{1}{\sec\theta-\tan\theta} \stackrel{?}{=} \sec\theta+\tan\theta$

$$= \frac{1}{\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{1}{\frac{1-\sin\theta}{\cos\theta}}$$

$$= \frac{\cos\theta}{1-\sin\theta}$$

$$= \frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

Because the left side had $(1-\sin\theta)$ as a factor, let's introduce it here.

$$= \frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta\,(1-\sin\theta)}$$

$$= \frac{1-\sin^2\theta}{\cos\theta\,(1-\sin\theta)}$$

$$= \frac{\cos^{\cancel{2}}\theta}{\cancel{\cos\theta}\,(1-\sin\theta)}$$

$$= \frac{\cos\theta}{1-\sin\theta}$$

$=$ ✓

4i $\dfrac{\sin\theta+\tan\theta}{1+\cos\theta} \stackrel{?}{=} \cot\theta$

$$= \frac{\sin\theta+\frac{\sin\theta}{\cos\theta}}{1+\cos\theta}$$

mult top and bottom by $\cos\theta$ to clear fractions inside of fractions

$$= \frac{\sin\theta\cos\theta+\sin\theta}{\cos\theta\,(1+\cos\theta)}$$

$$= \frac{\sin\theta\,(\cancel{1+\cos\theta})}{\cos\theta\,(\cancel{1+\cos\theta})}$$

$$= \frac{\sin\theta}{\cos\theta}$$

not equal for all θ where these expressions are defined – so not an identity!

4j $\cos^4 x-\sin^4 x \stackrel{?}{=} \cos 2x$

By double angle identity

$$= \cos^2 x-\sin^2 x$$

$$= (\cos^2 x-\sin^2 x)\underbrace{(\cos^2 x+\sin^2 x)}_{=1 \text{ by Pythag. identity}}$$

$$= \cos^2 x-\sin^2 x$$

$=$ ✓

4k $\dfrac{\tan t}{\tan^2 t-1} \stackrel{?}{=} \dfrac{1}{\tan t-\cot t}$

$$= \frac{\frac{\sin t}{\cos t}}{\frac{\sin^2 t}{\cos^2 t}-1}$$

$$= \frac{\sin t\cdot\cos t}{\sin^2 t-\cos^2 t}$$

$$= \frac{1}{\frac{\sin t}{\cos t}-\frac{\cos t}{\sin t}}$$

get common denom. to combine terms

$$= \frac{1}{\frac{\sin^2 t-\cos^2 t}{\cos t\cdot\sin t}}$$

reciprocate.

$$= \frac{\cos t\cdot\sin t}{\sin^2 t-\cos^2 t}$$

$=$ ✓

4l $\dfrac{\sin\theta+\cos\theta}{\sec\theta+\csc\theta} \stackrel{?}{=} \dfrac{\sin\theta}{\sec\theta}$

$$= \frac{\sin\theta+\cos\theta}{\frac{1}{\cos\theta}+\frac{1}{\sin\theta}}$$

$$= \frac{\sin\theta+\cos\theta}{\frac{\sin\theta+\cos\theta}{\cos\theta\cdot\sin\theta}}$$

$$= \frac{\cancel{\sin\theta+\cos\theta}}{1}\cdot\frac{\cos\theta\cdot\sin\theta}{\cancel{\sin\theta+\cos\theta}}$$

$$= \frac{\sin\theta}{\frac{1}{\cos\theta}}$$

$$= \sin\theta\cdot\cos\theta$$

$=$ ✓

4m

$$\frac{\sin\theta+\cos\theta}{\sec\theta+\csc\theta} \stackrel{?}{=} \frac{\sin\theta}{\sec\theta}$$

$$= \frac{\sin\theta+\cos\theta}{\frac{1}{\cos\theta}+\frac{1}{\sin\theta}}$$

$$= \frac{(\sin\theta+\cos\theta)}{\left(\frac{1}{\cos\theta}+\frac{1}{\sin\theta}\right)} \cdot \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cancel{(\sin\theta+\cos\theta)}\sin\theta\cos\theta}{\cancel{(\sin\theta+\cos\theta)}}$$

$$= \sin\theta\cos\theta$$

$$= \frac{\sin\theta}{\frac{1}{\cos\theta}}$$

$$= \sin\theta\cos\theta$$

✓

4n

$$\frac{\cos^4 x-\sin^4 x}{\sin x+\cos x} \stackrel{?}{=} \frac{1-\tan x}{1+\tan x}$$

$$= \frac{(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)}{\sin x+\cos x}$$

$$= \frac{\cancel{(\cos x+\sin x)}(\cos x-\sin x)\underline{(\cos^2 x+\sin^2 x)}}{\cancel{(\sin x+\cos x)}}$$

note, this is 1.

$$= (\cos x-\sin x)$$

$$= \frac{\left(1-\frac{\sin x}{\cos x}\right)}{\left(1+\frac{\sin x}{\cos x}\right)} \cdot \frac{\cos x}{\cos x}$$

$$= \frac{\cos x-\sin x}{\cos x+\sin x}$$

note, denominator is not always 1, making it not always equivalent to the expression at left, which shares the $(\cos x-\sin x)$ expression

$\neq$

This is not an identity!

(note $x=\frac{\pi}{6}$ provides a counterexample, among others)

✗