$-2\pi \quad -\frac{3\pi}{2} \quad -\pi \quad -\frac{\pi}{2} \quad -1 \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad 2\pi$

$y = \cos x$

$\left(\frac{\pi}{4}, 1\right) \qquad \frac{5\pi}{4}$

$-\frac{2\pi}{4} \quad -\frac{5\pi}{4} \quad -\frac{\pi}{4} \quad \frac{\pi}{4} \quad \frac{3\pi}{4} \quad \frac{7\pi}{4} \quad \frac{9\pi}{4}$

$\left(\frac{5\pi}{4}, -1\right)$

$y = \cos\left(x - \frac{\pi}{4}\right)$

$\left(\frac{\pi}{2}, 1\right) \qquad \frac{5\pi}{2}$

$-\frac{7\pi}{2} \quad -\frac{5\pi}{2} \quad -\frac{3\pi}{2} \quad -\frac{\pi}{2} \quad \frac{\pi}{2} \quad \frac{3\pi}{2} \quad \frac{5\pi}{2} \quad \frac{7\pi}{2} \quad \frac{9\pi}{2}$

$\left(\frac{5\pi}{2}, -1\right)$

$y = \cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$

$\left(\frac{\pi}{2}, 4\right) \qquad \frac{5\pi}{2}$

$-\frac{7\pi}{2} \quad -\frac{5\pi}{2} \quad -\frac{3\pi}{2} \quad -\frac{\pi}{2} \quad \frac{\pi}{2} \quad \frac{3\pi}{2} \quad \frac{5\pi}{2} \quad \frac{7\pi}{2} \quad \frac{9\pi}{2}$

$\left(\frac{5\pi}{2}, -4\right)$

$y = \cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$

$\left(\frac{5\pi}{2}, 4\right)$

$-\frac{7\pi}{2} \quad -\frac{5\pi}{2} \quad -\frac{\pi}{2} \quad \frac{\pi}{2} \quad \frac{3\pi}{2} \quad \frac{7\pi}{2} \quad \frac{9\pi}{2}$

$-\frac{3\pi}{2} \qquad \frac{5\pi}{2}$

$-\left(\frac{\pi}{2}\right) \quad \left(\frac{\pi}{2}, -4\right)$

$y = -4\cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$

At $x = \pi$,
$y = -4\cos\left(-\frac{\pi}{4}\right) = -4\left(-\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$

At $x = 3\pi$,
$y = -4\cos\frac{5\pi}{4} = -4\left(-\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$

$\left(-\pi, \frac{\sqrt{2}}{2}\right) \quad -\frac{\pi}{2} \qquad \left(\frac{5\pi}{2}, 4\right)$

$\frac{\pi}{2} \quad \frac{3\pi}{2} \qquad \left(3\pi, \frac{\sqrt{2}}{2}\right)$

$\frac{5\pi}{2}$

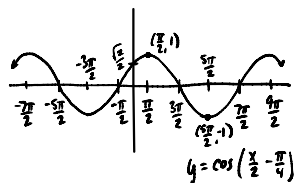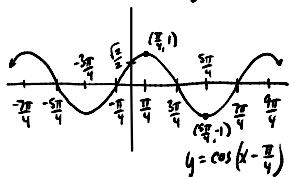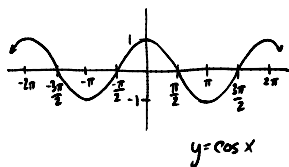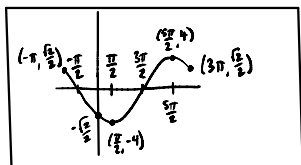$-\left(\frac{\sqrt{2}}{2}\right) \quad \left(\frac{\pi}{2}, -4\right)$

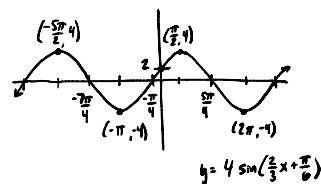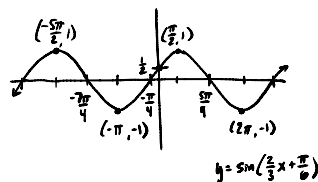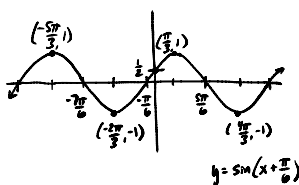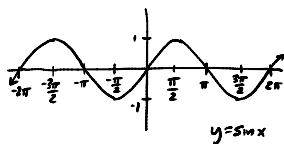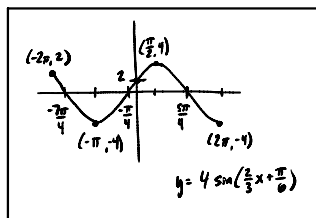$y = -4\cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$

on $[-\pi, 3\pi]$

$-2\pi \quad -\frac{3\pi}{2} \quad -\pi \quad -\frac{\pi}{2} \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad 2\pi$

-1

$y = \sin x$

$\left(-\frac{5\pi}{3}, 1\right) \qquad \left(\frac{\pi}{3}, 1\right)$

$-\frac{7\pi}{6} \qquad -\frac{\pi}{6} \qquad \frac{5\pi}{6}$

$\left(-\frac{2\pi}{3}, -1\right) \qquad \left(\frac{4\pi}{3}, -1\right)$

$y = \sin\left(x + \frac{\pi}{6}\right)$

$\left(-\frac{5\pi}{2}, 1\right) \qquad \left(\frac{\pi}{2}, 1\right)$

$-\frac{7\pi}{4} \qquad -\frac{\pi}{4} \qquad \frac{5\pi}{4}$

$(-\pi, -1) \qquad (2\pi, -1)$

$y = \sin\left(\frac{2}{3}x + \frac{\pi}{6}\right)$

$\left(-\frac{5\pi}{2}, 4\right) \qquad \left(\frac{\pi}{2}, 4\right)$

2

$-\frac{7\pi}{4} \qquad -\frac{\pi}{4} \qquad \frac{5\pi}{4}$

$(-\pi, -4) \qquad (2\pi, -4)$

$y = 4\sin\left(\frac{2}{3}x + \frac{\pi}{6}\right)$

At $x = -2\pi$
$y = 4\sin\left(-\frac{4\pi}{3} + \frac{\pi}{6}\right) = 4\sin\left(-\frac{7\pi}{6}\right) = 2$
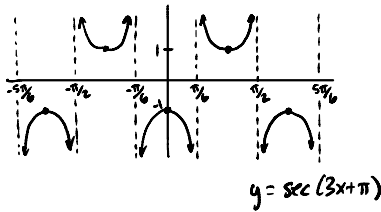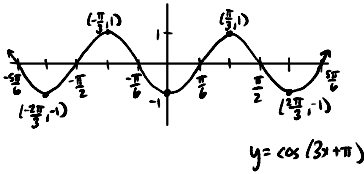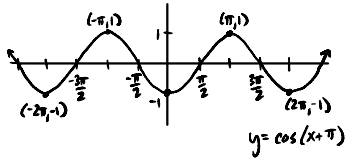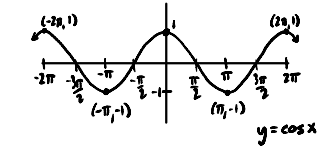
At $x = 2\pi$
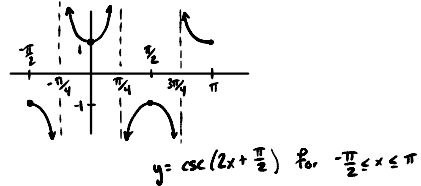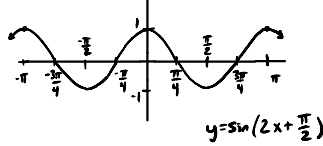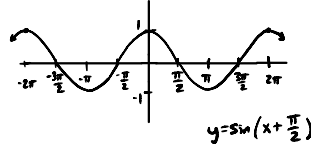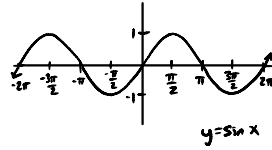$y = 4\sin\left(\frac{4\pi}{3} + \frac{\pi}{6}\right) = 4\sin\left(\frac{3\pi}{2}\right) = -4$

$(-2\pi, 2) \qquad \left(\frac{\pi}{2}, 4\right)$

2

$-\frac{7\pi}{4} \qquad -\frac{\pi}{4} \qquad \frac{5\pi}{4}$

$(-\pi, -4) \qquad (2\pi, -4)$

$y = 4\sin\left(\frac{2}{3}x + \frac{\pi}{6}\right)$

(3c) $y = -2\sec(3x+\pi)$ on $\left[-\frac{\pi}{3}, \frac{2\pi}{3}\right]$

$y = \cos x$

$y = \cos(x+\pi)$

$y = \cos(3x+\pi)$

$y = \sec(3x+\pi)$

$y = 2\sec(3x+\pi)$

$y = -2\sec(3x+\pi)$

$y = -2\sec(3x+\pi)$

(3d) $y = 2\csc\left(2x+\frac{\pi}{2}\right)$ for $-\frac{\pi}{2} \le x \le \pi$

$y = \sin x$

$y = \sin\left(x+\frac{\pi}{2}\right)$

$y = \sin\left(2x+\frac{\pi}{2}\right)$
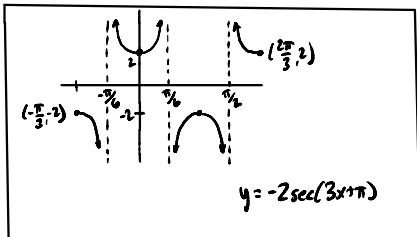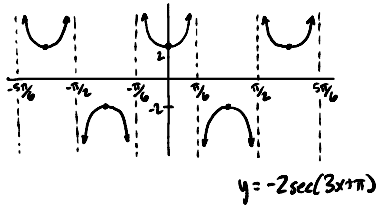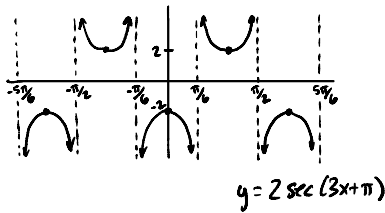
$y = \csc\left(2x+\frac{\pi}{2}\right)$ for $-\frac{\pi}{2} \le x \le \pi$

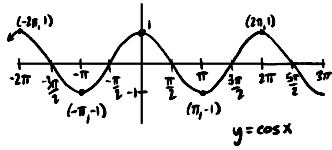$y = \csc\left(2x+\frac{\pi}{2}\right)$ for $-\frac{\pi}{2} \le x \le \pi$
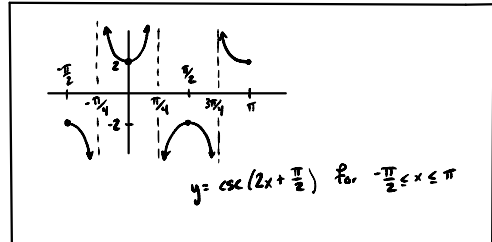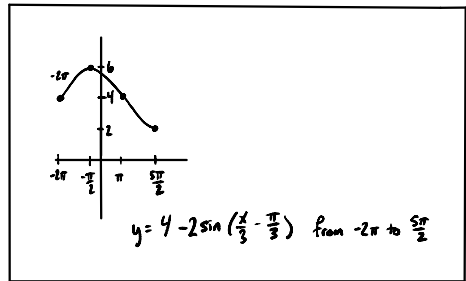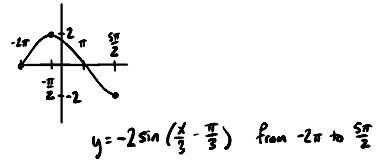
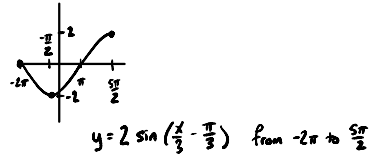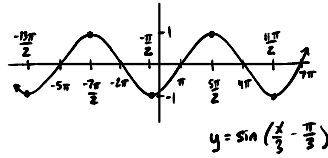At $x = \frac{-\pi}{3}$

$y = -2\sec(0) = \frac{-2}{\cos(0)} = -2$
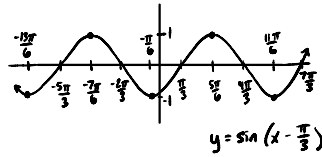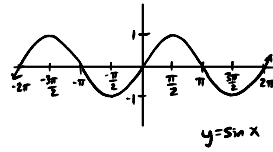
At $x = \frac{2\pi}{3}$

$y = -2\sec(3\pi) = \frac{-2}{\cos(3\pi)} = 2$

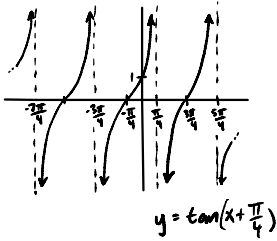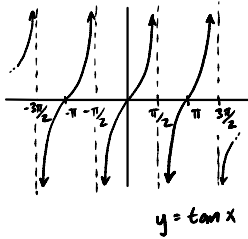③e $y = 5\cos\left(3x + \frac{\pi}{2}\right)$ on $[0, \pi]$

$(-2\pi, 1)$ $(2\pi, 1)$

$(-\pi, -1)$ $(\pi, -1)$

$y = \cos x$

$y = \cos\left(x + \frac{\pi}{2}\right)$

$y = \cos\left(x + \frac{\pi}{2}\right)$ on $[0, \pi]$

$y = 5\cos\left(x + \frac{\pi}{2}\right)$ on $[0, \pi]$

③f $y = 4 - 2\sin\left(\frac{x}{3} - \frac{\pi}{3}\right)$ from -2π to $\frac{5\pi}{2}$

$y = \sin x$

$y = \sin\left(x - \frac{\pi}{3}\right)$

$y = \sin\left(\frac{x}{3} - \frac{\pi}{3}\right)$

$y = 2\sin\left(\frac{x}{3} - \frac{\pi}{3}\right)$ from -2π to $\frac{5\pi}{2}$

$y = -2\sin\left(\frac{x}{3} - \frac{\pi}{3}\right)$ from -2π to $\frac{5\pi}{2}$

$y = 4 - 2\sin\left(\frac{x}{3} - \frac{\pi}{3}\right)$ from -2π to $\frac{5\pi}{2}$

③g $y = \tan\left(x + \frac{\pi}{4}\right)$ on $[-\pi, 2\pi]$

$y = \tan x$

$y = \tan\left(x + \frac{\pi}{4}\right)$

At $x = -\pi$ | At $x = 2\pi$

$y = \tan\left(-\frac{3\pi}{4}\right) = 1$ | $y = \tan\left(\frac{9\pi}{4}\right) = 1$

$(-\pi, 1)$ $(2\pi, 1)$

$y = \tan\left(x + \frac{\pi}{4}\right)$ on $[-\pi, 2\pi]$

③h $y = -2\sin\left(x - \frac{3\pi}{2}\right)$ on $\left[-\frac{\pi}{2}, 2\pi\right]$

$y = \sin x$

$y = \sin\left(x - \frac{3\pi}{2}\right)$
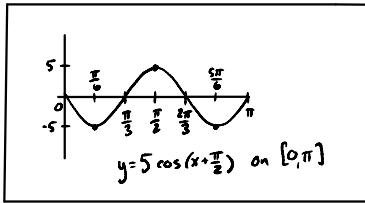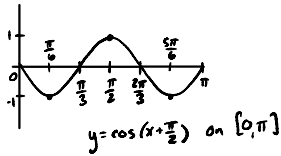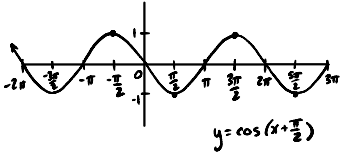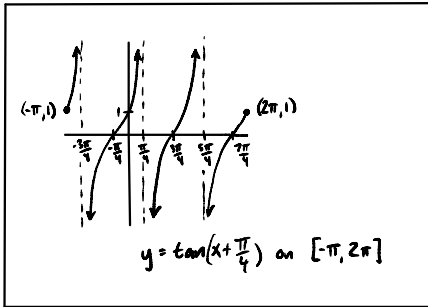
$y = 2\sin\left(x - \frac{3\pi}{2}\right)$ on $\left[-\frac{\pi}{2}, 2\pi\right]$

$y = -2\sin\left(x - \frac{3\pi}{2}\right)$ on $\left[-\frac{\pi}{2}, 2\pi\right]$

③i) $y = \cot\left(x - \frac{\pi}{6}\right)$ from $-\pi$ to 2π

$y = \cot x$

Note
$\cot\left(-\frac{\pi}{6}\right) = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

$y = \cot\left(x - \frac{\pi}{6}\right)$

At $x = -\pi$, $\quad y = \cot\left(\frac{-7\pi}{6}\right)$

$\qquad = \dfrac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

At $x = 2\pi$

$y = \cot\left(\frac{11\pi}{6}\right) = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

$(-\pi, -\sqrt{3})$ $\quad$ $-\sqrt{3}$ $\quad$ $(2\pi, -\sqrt{3})$

$y = \cot\left(x - \frac{\pi}{6}\right)$

③j) $y = -\frac{1}{3}\cos\left(\frac{x}{2} + \frac{\pi}{4}\right)$ on $[-2\pi, 4\pi]$

$y = \cos x$

$y = \cos\left(x + \frac{\pi}{4}\right)$

$y = \cos\left(\frac{x}{2} + \frac{\pi}{4}\right)$
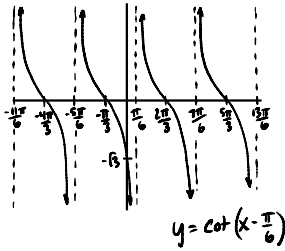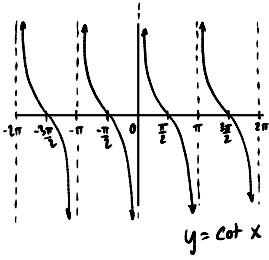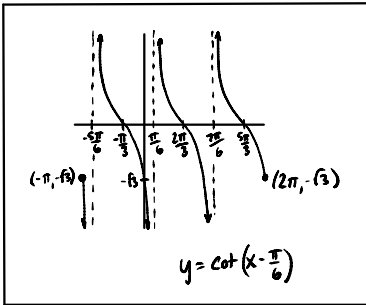
$y = \frac{1}{3}\cos\left(\frac{x}{2} + \frac{\pi}{4}\right)$

$y = -\frac{1}{3}\cos\left(\frac{x}{2} + \frac{\pi}{4}\right)$
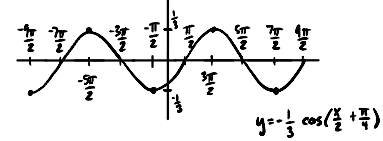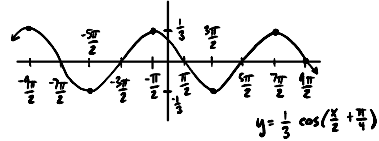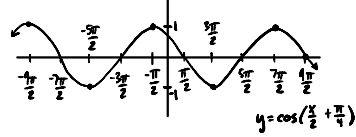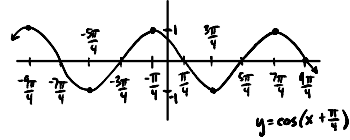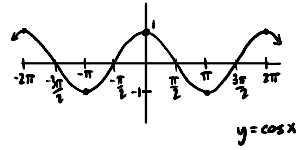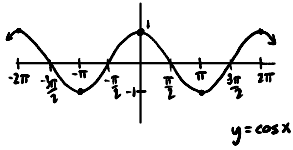
At $x = -2\pi$

$y = -\frac{1}{3}\cos\left(-\frac{2\pi}{4}\right) = \left(-\frac{1}{3}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{6}$

At $x = 4\pi$

$y = -\frac{1}{3}\cos\left(\frac{9\pi}{4}\right) = \left(-\frac{1}{3}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{2}}{6}$

$\left(-2\pi, \frac{\sqrt{2}}{6}\right)$ $\qquad$ $\left(4\pi, \frac{-\sqrt{2}}{6}\right)$

$y = -\frac{1}{3}\cos\left(\frac{x}{2} + \frac{\pi}{4}\right)$

(3k) $y = -\sec \frac{x}{2}$ for $-3\pi < x < 3\pi$

$y = \cos x$

$y = \cos \frac{x}{2}$

$y = \sec \frac{x}{2}$ for $-3\pi < x < 3\pi$