

$$\textcircled{a} \quad 4|x-1|-7=13$$

$$4|x-1|=20$$

$$|x-1|=5$$

$$x-1=5 \quad \text{OR} \quad x-1=-5$$

$$\boxed{x=6 \quad \text{OR} \quad x=-4}$$

$$\textcircled{b} \quad \left| \frac{x-2}{3} \right| = 4$$

$$\frac{x-2}{3} = 4 \quad \text{OR} \quad \frac{x-2}{3} = -4$$

$$x-2=12 \quad \text{OR} \quad x-2=-12$$

$$\boxed{x=14 \quad \text{OR} \quad x=-10}$$

$$\textcircled{c} \quad 4x^2+8x=-3$$

$$4x^2+8x+3=0$$

$$(2x+1)(2x+3)=0$$

$$2x+1=0 \quad \text{OR} \quad 2x+3=0$$

$$2x=-1 \quad \text{OR} \quad 2x=-3$$

$$\boxed{x=-\frac{1}{2} \quad \text{OR} \quad x=-\frac{3}{2}}$$

$$\textcircled{d} \quad 2x^2=5x+12$$

$$2x^2-5x-12=0$$

$$(2x+3)(x-4)=0$$

$$2x+3=0 \quad \text{OR} \quad x-4=0$$

$$2x=-3 \quad \text{OR} \quad x=4$$

$$\boxed{x=-\frac{3}{2} \quad \text{OR} \quad x=4}$$

$$\textcircled{e} \quad \sqrt{33-2x}=x+1$$

↘ check ans!

$$33-2x=x^2+2x+1$$

$$x^2+4x-32=0$$

$$(x+8)(x-4)=0$$

$$x+8=0 \quad \text{OR} \quad x-4=0$$

$$x=-8 \quad \text{OR} \quad x=4$$

$$\boxed{x=4}$$

$$\textcircled{f} \quad 7 = \sqrt{39+3x} - x$$

$$x+7 = \sqrt{39+3x}$$

↘ Check Ans!

$$x^2+14x+49 = 39+3x$$

$$x^2+11x+10=0$$

$$(x+1)(x+10)=0$$

$$x+1=0 \quad \text{OR} \quad x+10=0$$

$$\boxed{x=-1 \quad \text{OR} \quad x=-10}$$

$$\textcircled{g} \quad 3^x+2 \cdot 3^x=1$$

$$3 \cdot 3^x=1$$

$$3^x=\frac{1}{3}$$

$$\log_3 3^x = \log_3 \frac{1}{3}$$

$$\boxed{x=-1}$$

$$\textcircled{h} \quad \sqrt{x-5} + \sqrt{x} = 5$$

$$\sqrt{x-5} = 5 - \sqrt{x}$$

↘ check ans!

$$x-5 = 25 - 10\sqrt{x} + x$$

$$-30 = -10\sqrt{x}$$

$$3 = \sqrt{x}$$

$$\boxed{x=9} \quad \checkmark$$

$$\textcircled{i} \quad \sqrt{x+\sqrt{x-3}} = 3$$

$$x+\sqrt{x-3} = 9$$

↘ check ans!

$$\sqrt{x-3} = 9-x$$

$$x-3 = 81 - 18x + x^2$$

$$x^2 - 19x + 84 = 0$$

$$(x-7)(x-12) = 0$$

$$x-7=0 \quad \text{OR} \quad x-12=0$$

$$\boxed{x=7 \quad \text{OR} \quad x=12}$$

$$\textcircled{j} \quad x^3+6x^2=2x+12$$

$$x^3+6x^2-2x-12=0$$

$$x^2(x+6)-2(x+6)=0$$

$$(x^2-2)(x+6)=0$$

$$x^2-2=0 \quad \text{OR} \quad x+6=0$$

$$x^2=2 \quad \text{OR} \quad x=-6$$

$$\boxed{x=\pm\sqrt{2} \quad \text{OR} \quad x=-6}$$

$$\textcircled{k} \quad 2x-5\sqrt{x}+2=0$$

$$2x+2=5\sqrt{x}$$

$$4x^2+8x+4=25x$$

$$4x^2-17x+4=0$$

$$(x-4)(4x-1)=0$$

$$x-4=0 \quad \text{OR} \quad 4x-1=0$$

$$\boxed{x=4 \quad \text{OR} \quad x=\frac{1}{4}}$$

$$\textcircled{l} \quad 3x^{2/3} - 5x^{1/3} - 2 = 0$$

$$3(x^{1/3})^2 - 5(x^{1/3}) - 2 = 0$$

↘ Let $u=x^{1/3}$

$$3u^2 - 5u - 2 = 0$$

$$(3u+1)(u-2) = 0$$

$$3u+1=0 \quad \text{OR} \quad u-2=0$$

$$3u=-1 \quad \text{OR} \quad u=2$$

$$u=-\frac{1}{3} \quad \text{OR} \quad u=2$$

$$x^{1/3} = -\frac{1}{3} \quad \text{OR} \quad x^{1/3} = 2$$

$$\boxed{x = -\frac{1}{27} \quad \text{OR} \quad x = 8}$$

$$\textcircled{m} \quad 2x^2 - 11x^{\frac{1}{2}} + 5 = 0$$

$$2(x^{\frac{1}{2}})^2 - 11(x^{\frac{1}{2}}) + 5 = 0$$

↘ Let $u=x^{\frac{1}{2}}$

$$2u^2 - 11u + 5 = 0$$

$$(2u-1)(u-5) = 0$$

$$2u-1=0 \quad \text{OR} \quad u-5=0$$

$$2u=1 \quad \text{OR} \quad u=5$$

$$u=\frac{1}{2} \quad \text{OR} \quad u=5$$

$$x^{\frac{1}{2}} = \frac{1}{2} \quad \text{OR} \quad x^{\frac{1}{2}} = 5$$

$$\boxed{x = \frac{1}{4} \quad \text{OR} \quad x = 25}$$

(1) $\log_2 x^2 = \log_2^2 x$
 $2(\log_2 x) = (\log_2 x)^2$
 $(\log_2 x)^2 - 2(\log_2 x) = 0$

$u^2 - 2u = 0$ \leftarrow Let $u = \log_2 x$

$u(u-2) = 0$

$u = 0$ or $u - 2 = 0$

$u = 2$

$\log_2 x = 0$ OR $\log_2 x = 2$

$x = 2^0$ OR $x = 2^2$

$x = 1$ OR $x = 4$

(2) $\log_3(x+6) = 3 - \log_3 x$
 $\log_3(x+6) + \log_3 x = 3$

$\log_3 x(x+6) = 3$

$x(x+6) = 3^3$

$x(x+6) = 27$

$x^2 + 6x = 27$

$x^2 + 6x - 27 = 0$

$(x+9)(x-3) = 0$

$x+9 = 0$ OR $x-3 = 0$

$x = -9$ OR $x = 3$

domain issue!

(3) $\log_4(x+12) - \log_4(x-3) = 2$
 $\log_4\left(\frac{x+12}{x-3}\right) = 2$

$\frac{x+12}{x-3} = 4^2 \rightarrow \frac{x+12}{x-3} = 16$

$x+12 = 16(x-3)$ with $x \neq 3$

$x+12 = 16x - 48$

$15x - 60 = 0$

$15x = 60$

$x = 4$

(4) $\frac{1}{2} \log_4(2x+1) = \log_4 3$

$\log_4(2x+1)^{1/2} = \log_4 3$

$\sqrt{\log_4(2x+1)^{1/2}} = \sqrt{\log_4 3}$

$\sqrt{2x+1} = 3$

$2x+1 = 9$

$2x = 8$

$x = 4$

Check answers in case of domain issues!

(5) $\log_2(x+5) - \log_2 x = \log_2 4$

$\log_2 \frac{x+5}{x} = \log_2 4$

$2^{\log_2 \frac{x+5}{x}} = 2^{\log_2 4}$

$\frac{x+5}{x} = 4$

$x+5 = 4x$

$3x = 5$

$x = \frac{5}{3}$

Check, but it's ok.

(6) $\log_4(x+3) + \log_4(x-3) = 2$

$\log_4(x+3)(x-3) = 2$

$\log_4(x^2-9) = 2$

$4^{\log_4(x^2-9)} = 4^2$

$x^2-9 = 16$

$x^2 = 25$

$x = \pm 5$

But -5 causes inputs to the logs in orig. eqn to be domain issues, so...

$x = 5$

(7) $\frac{2}{x-5} - \frac{1}{x+3} = \frac{8}{x^2-2x-15}$

$\frac{2}{x-5} - \frac{1}{x+3} = \frac{8}{(x+3)(x-5)}$

$\frac{2}{x-5} \cdot \frac{(x+3)}{(x+3)} - \frac{1}{x+3} \cdot \frac{(x-5)}{(x-5)} = \frac{8}{(x+3)(x-5)}$

$2(x+3) - (x-5) = 8$

$2x+6-x+5 = 8$

$x+11 = 8$

$x = -3$

but this causes are of the denom. in original eqn to be zero, so...

no solution

Be aware of domain issues! (no zeros in denom.)