(1a)

$3x - 7 = 0$

$3x = 7$

$\boxed{x = \frac{7}{3}}$

(1b)

$\frac{5x^3+2}{7} = 6$

$5x^3+2 = 42$

$5x^3 = 40$

$x^3 = 8$

$\boxed{x = 2}$

(1c)

$4\left(\frac{\sqrt[3]{x+1}}{7} + 2\right)^5 = \frac{1}{8}$

$\left(\frac{\sqrt[3]{x+1}}{7} + 2\right)^5 = \frac{1}{32}$

$\frac{\sqrt[3]{x+1}}{7} + 2 = \frac{1}{2}$

$\frac{\sqrt[3]{x+1}}{7} = \frac{-3}{2}$

$\sqrt[3]{x+1} = \frac{-21}{2}$

$x+1 = -\frac{9261}{8}$

$\boxed{x = -\frac{9269}{8}}$

(1d)

$\frac{1}{3-\frac{1}{x}} = \frac{4}{11}$

$3 - \frac{1}{x} = \frac{11}{4}$

$-\frac{1}{x} = \frac{-1}{4}$

$\frac{1}{x} = \frac{1}{4}$

$\boxed{x = 4}$

(1e)

$\left(\sqrt[3]{x} + 5\right)^{-1} = 2$

$\sqrt[3]{x} + 5 = \frac{1}{2}$

$\sqrt[3]{x} = \frac{-9}{2}$

$\boxed{x = \frac{-729}{8}}$

(1f)

$2x^4 - 8 = 0$

$2x^4 = 8$

$x^4 = 4$

$x = \pm\sqrt[4]{4}$

technically this last step didn't result from the application of a function to both sides, but should be a reasonable step to take, regardless.

(1g)

$\log_6(x^2+11) = 2$

note: we applied 6^x to both sides, since that is the inverse of $\log_6 x$:

$\cancel{6}^{\cancel{\log_6}(x^2+11)} = 6^2$

$x^2 + 11 = 36$

$x^2 = 25$

$\boxed{x = \pm 5}$

(1h)

$2^{x^3-7} = 5$

Note: we applied $\log_2 x$ to both sides, since that is the inverse to 2^x:

$\cancel{\log_2} \cancel{2}^{x^3-7} = \log_2 5$

$x^3 - 7 = \log_2 5$

$x^3 = 7 + \log_2 5$

note, we place the 7 to the left of the log so there is no confusion with reading this as $\log_2(5+7)$

$\boxed{x = \sqrt[3]{7+\log_2 5}}$

(2a) $x(x^2+3)-x=2x-8$ — expand

$x^3+3x-x=2x-8$ — collect like terms

$x^3+2x=2x-8$ — subtract $2x$ from both sides

$x^3=-8$ — now just "take off the socks"

$x=-2$

(2b) $z=36z^3$ — be careful here! If you divide both sides by z you have to consider $z=0$ separately! (It's a solution.) Safer perhaps, to just get everything to one side by subtracting $36z^3$ from both sides.

$z-36z^3=0$

$z(1-36z^2)=0$

$z(1-6z)(1+6z)=0$

Given the zero on the right, factoring sets us up to solve with the zero-product property

$\boxed{z=0}$ OR $1-6z=0$ OR $1+6z=0$

$-6z=-1$ $6z=-1$

$\boxed{z=\frac{1}{6}}$ $\boxed{z=\frac{-1}{6}}$

So $\boxed{z=0 \text{ OR } z=\pm\frac{1}{6}}$

(2c)

$2x^3+x^2-2x-1=0$

$x^2(2x+1)-(2x+1)=0$

$(2x+1)(x^2-1)=0$

$(2x+1)(x+1)(x-1)=0$

$2x+1=0$ OR $x+1=0$ OR $x-1=0$

$2x=-1$ $\boxed{x=-1}$ $\boxed{x=1}$

$\boxed{x=\frac{-1}{2}}$

So $\boxed{x=-\frac{1}{2} \text{ or } x=\pm 1}$

(2d) $a^{2/3}-3a^{1/3}-10=0$ — Recognizing that $a^{2/3}$ is the square of $a^{1/3}$ we try a substitution...

Let $u=a^{1/3}$

$u^2-3u-10=0$ — Seeing the zero on the right side immediately gives us the option to try solving with the zero-product property

$(u-5)(u+2)=0$

$u-5=0$ OR $u+2=0$

$u=5$ OR $u=-2$ — don't forget — we are solving for a, not u!

There wasn't even enough left where we needed to employ "socks and shoes here! (maybe just "socks") ☺

$a^{\frac{1}{3}}=5$ OR $a^{\frac{1}{3}}=-2$

also here..

$\boxed{a=125 \text{ OR } a=-8}$

2e

$$(a^2-1)^2-(a^2-1)-2=0$$

Seeing both (a^2-1) and its square, we might try a substitution...

Let $u=a^2-1$

$$u^2-u-2=0$$

seeing the zero on the right we notice we might be able to use the zero-product property

$$(u-2)(u+1)=0$$

$$u-2=0 \quad \underline{\underline{OR}} \quad u+1=0$$

"socks" only ☺

$$u=2 \quad \underline{\underline{OR}} \quad u=-1$$

remember you need a, not u.

$$a^2-1=2 \quad \underline{\underline{OR}} \quad a^2-1=-1$$

$$a^2=3 \quad \underline{\underline{OR}} \quad a^2=0$$

"Socks and shoes" twice...

$$\boxed{a=\pm\sqrt{3} \quad \underline{\underline{OR}} \quad a=0}$$

2f

$$(6x^3+x^2-35x)(49-x^4)=0$$

with a partial factorization already, let's finish the job!

$$x(6x^2+x-35)(7-x^2)(7+x^2)=0$$

both of these are irreducible for polynomials with integer coefficients by Eisenstein's Criterion, $p=7$. Of course notice that $7-x^2=(\sqrt{7}+x)(\sqrt{7}-x)$ too! ☺

$$x(3x-7)(2x+5)(7-x^2)(7+x^2)=0$$

... then use the zero-product property

$\boxed{x=0}$ $\underline{\underline{OR}}$ $3x-7=0$ $\underline{\underline{OR}}$ $7-x^2=0$ $\underline{\underline{OR}}$ $7+x^2=0$

$3x=7$

$\boxed{x=\frac{7}{3}}$

$-x^2=-7$

$x^2=7$

$\boxed{x=\pm\sqrt{7}}$

$x^2=-7$

$\underline{\underline{\text{impossible}}}$!

(no solution for this one part)

several places to use "socks and shoes"...

So, $\boxed{x=0, \frac{7}{3}, \text{ or } \pm\sqrt{7}}$

2g

$$\log_{10}(|x|+1)-\log_{10}(|x|+2)=0$$

If you stop and think before trying to reduce the number of x variables seen you can actually solve this in $\underline{\text{one}}$ step. Do you see how?

$$\log_{10}\left(\frac{|x|+1}{|x|+2}\right)=0$$

$$\frac{|x|+1}{|x|+2}=1$$

This is the result of adding a well-chosen value of zero

$$\frac{|x|+1}{|x|+2}=\frac{|x|+1\overbrace{+1-1}^{=0}}{|x|+2}$$

$$=\frac{|x|+2-1}{|x|+2}$$

$$=\frac{|x|+2}{|x|+2}-\frac{1}{|x|+2}=1-\frac{1}{|x|+2}$$

$$1-\frac{1}{|x|+2}=1$$

socks and shoes

$$\frac{-1}{|x|+2}=0 \quad \text{impossible!}$$

(the numerator is not zero)

$\boxed{\text{No Solution!}}$

(2h) $5\sqrt[3]{x^2} - 4\sqrt[3]{x} = 1$

Let: $u = \sqrt[3]{x}$

$5u^2 - 4u = 1$

$5u^2 - 4u - 1 = 0$

$(5u+1)(u-1) = 0$

$5u+1 = 0$ OR $u-1 = 0$

$5u = -1$ $\quad$ $u = 1$

$u = \frac{-1}{5}$

$u = -\frac{1}{5}$ OR $u = 1$

↓

$\sqrt[3]{x} = -\frac{1}{5}$ OR $\sqrt[3]{x} = 1$

$x = \frac{-1}{125}$ $\quad$ $x = 1$

So $\boxed{x = \frac{-1}{125} \text{ OR } x = 1}$

(2i) $x^3 + 2x^2 - 3ax = 6a$

$x^3 + 2x^2 - 3ax - 6a = 0$

$x^2(x+2) - 3a(x+2) = 0$

$(x+2)(x^2 - 3a) = 0$

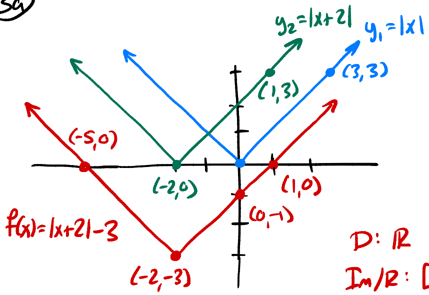
$x+2 = 0$ OR $x^2 - 3a = 0$

$x = -2$ $\quad$ $x^2 = 3a$

$x = \pm\sqrt{3a}$

So $\boxed{x = -2 \text{ or } \pm\sqrt{3a}}$

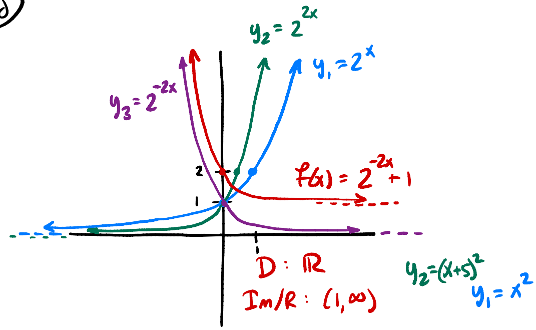
3a



D: $\mathbb{R}$

Im/R: $[-3,\infty)$

3d



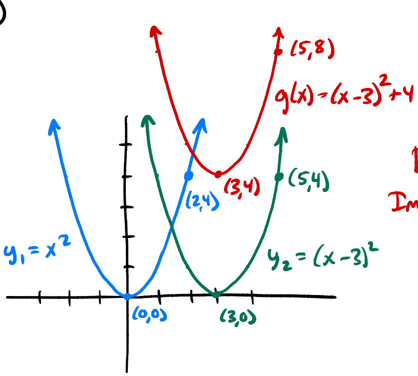
D: $\mathbb{R}$

Im/R: $(1,\infty)$

$y_2=(x+5)^2$

$y_1=x^2$

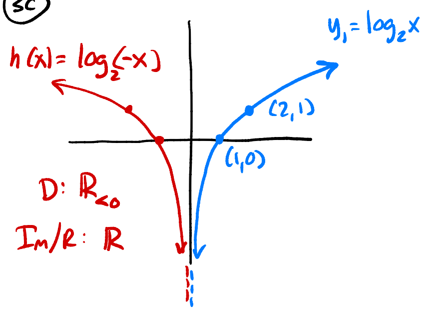
3b



D: $\mathbb{R}$

Im/R: $[4,\infty)$

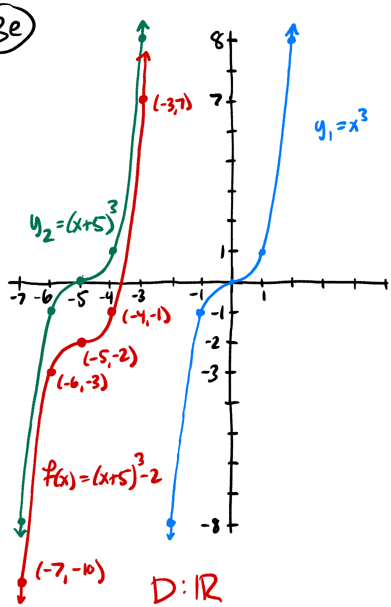
3c



D: $\mathbb{R}_{<0}$

Im/R: $\mathbb{R}$

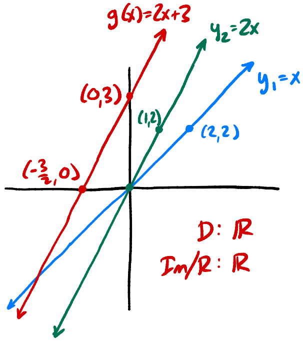
3e



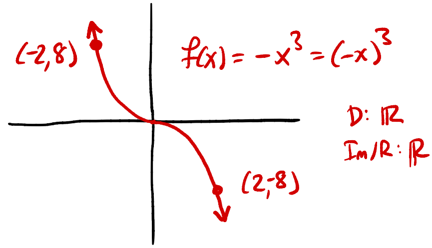
D: $\mathbb{R}$

Im/R: $\mathbb{R}$

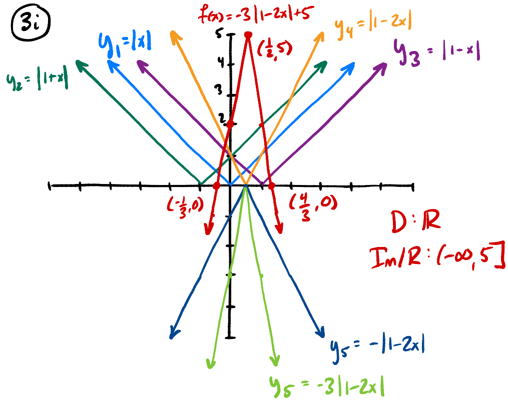
3f



3g and 3h



3i

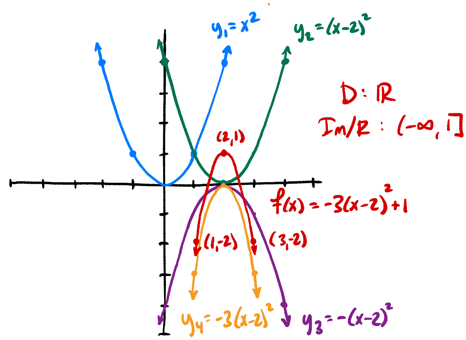


3j

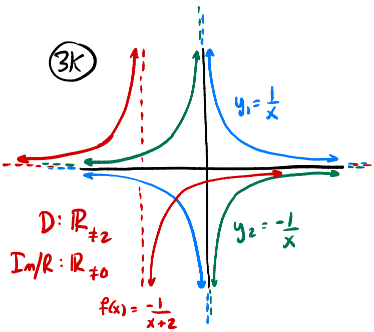
Note we can simplify things a little bit if we observe $(2-x)^2=(x-2)^2$ ...

So we plot $f(x)=-3(x-2)^2+1$ below

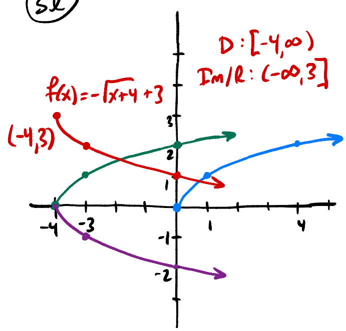
[we save ourselves one transformation]



3K



3l



3m