(a) We seek the inverse of the linear function:

$$f(x) = -\frac{2}{3}x + 4$$

by socks and shoes

$$f^{-1}(x) = \frac{x-4}{-\frac{2}{3}}$$

$$= (x-4)\left(-\frac{3}{2}\right)$$

$$= -\frac{3}{2}x + 6$$

(b) We seek the inverse of the linear function:

$$f(x) = 6x - \frac{1}{2}$$

by socks and shoes

$$f^{-1}(x) = \frac{x+\frac{1}{2}}{6}$$

$$= \left(x+\frac{1}{2}\right)\left(\frac{1}{6}\right)$$

$$= \frac{1}{6}x + \frac{1}{12}$$

(c) $f(x) = 7$ has no inverse as its graph does not pass the horizontal line test

There are multiple inputs that lead to the same output - so its not injective

(d) $f(x) = \frac{x+4}{x-3}$ ← We seek the inverse of a Mobius transformation

For any (x,y) on the graph of f, we know $y = \frac{x+4}{x-3}$

But then, for any (x,y) on the graph of f^{-1}, $x = \frac{y+4}{y-3}$ ← So we solve this for y to find a formula for $f^{-1}(x)$

$$x = \frac{y+4}{y-3}$$

$$x(y-3) = y+4$$

$$xy - 3x = y + 4$$

$$xy - y = 3x + 4$$

$$y(x-1) = 3x+4$$

$$y = \frac{3x+4}{x-1}$$

Thus, $\boxed{f^{-1}(x) = \frac{3x+4}{x-1}}$

(e) $f(x) = \frac{5x-3}{2x+1}$ ← We seek the inverse of a Mobius transformation

For any (x,y) on the graph of f, we know $y = \frac{5x-3}{2x+1}$

But then for any (x,y) on the graph of f^{-1}, $x = \frac{5y-3}{2y+1}$ ← So we solve this for y to find a formula for $f^{-1}(x)$

$$x = \frac{5y-3}{2y+1}$$

$$x(2y+1) = 5y-3$$

$$2xy + x = 5y - 3$$

$$2xy - 5y = -x - 3$$

$$y(2x-5) = -x-3$$

$$y = \frac{-x-3}{2x-5}$$

So $\boxed{f^{-1}(x) = \frac{-x-3}{2x-5}}$

(f) $f(x) = \frac{x+6}{3x-4}$ ← We seek the inverse of a Mobius transformation

For any (x,y) on the graph of $f(x)$, we know $y = \frac{x+6}{3x-4}$

So for any (x,y) on the graph of $f^{-1}(x)$ we know $x = \frac{y+6}{3y-4}$ ← So we solve this for y to find a formula for $f^{-1}(x)$

$$x = \frac{y+6}{3y-4}$$

$$x(3y-4) = y+6$$

$$3xy - 4x = y + 6$$

$$3xy - y = 4x + 6$$

$$y(3x-1) = 4x+6$$

$$y = \frac{4x+6}{3x-1}$$

So, $\boxed{f^{-1}(x) = \frac{4x+6}{3x-1}}$