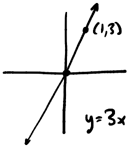
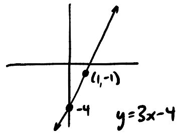
(2a) $f(x) = 3x - 4$

Compositional Chain:
$x \to 3x \to x-4$



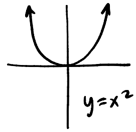
start with the scaling $y=x$ by a factor of 3



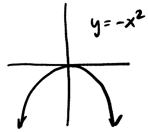
then shift down (a.k.a. vertically translate down) by 4 units.

(2b) $f(x) = -x^2+2$

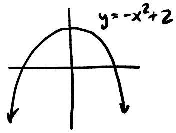
Compositional Chain:
$x \to x^2 \to -x \to x+2$



Start with the graph of $y=x^2$



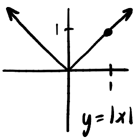
then reflect it over the x-axis



then vertically translate it up 2 units

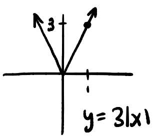
(2c) $f(x) = -3|x| - 2$

Compositional Chain:
$x \to |x| \to 3x \to -x \to x-2$

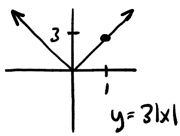


Start with the graph of $|x|$. equivalently, "fold up" the left, negative part of $y=x$

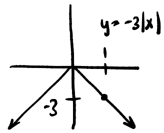
note we also indicate the coordinates of one easy-to-find point to give an idea of scale



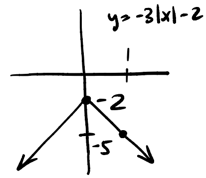
then vertically scale by a factor of 3



OR, more conveniently, change the scale on the y-axis, as this allows us to draw a picture very similar to the first one. This is particularly useful when scaling by a very large or very small factor.



then reflect (vertically) over the x-axis

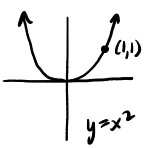


then vertically translate down 2 units

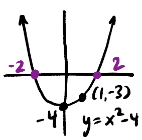
(2d) $f(x) = |x^2-4|$

Compositional Chain:

$x \to x^2 \to x-4 \to |x|$



first we graph $y=x^2$, marking an easy-to-find pt for scale.



then we vertically translate down 4 units, tracking where our marked pt went. However, note 2 new interesting points show up (i.e. the x-intercepts)

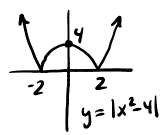
We can find these x-intercepts by solving

$x^2-4=0$

$(x+2)(x-2)=0$

$x=\pm 2$ ← zero product property

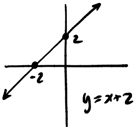
as the x and y intercepts establish the scale, we drop the "extra point" in the next picture

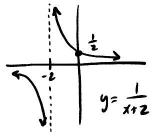


Finally we "fold up" the negative part of the graph to account for the affect of the absolute value.

(2e) $f(x) = \frac{1}{x+2}$

Compositional Chain
$x \to x+2 \to \frac{1}{x}$





we start with the graph of $y=x+2$ (i.e the graph of x shifted up 2 units)

then we apply the "non-rigid" transformation of reciprocation (where small magnitude y-values become big magnitude y-values (of the same sign) and vice-versa

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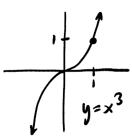
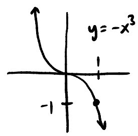
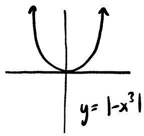
(2f) $f(x) = |-x^3|$

Compositional Chain : $x \to x^3 \to -x \to |x|$







We start with the graph of $y=x^3$

then reflect over the x-axis

then fold up the negative part of the graph to account for the absolute value

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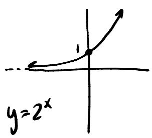
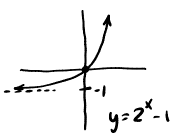
(2g) $f(x) = 2^x - 1$

Compositional Chain : $x \to 2^x \to x-1$

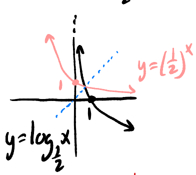


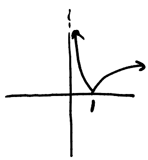


we start with the graph of $y=2^x$

then vertically translate downward one unit

(2h) $f(x) = \left|\log_{\frac{1}{2}}(x)\right|$





start with the graph of $\log_{\frac{1}{2}}(x)$ which – if it helps – is the inverse of $(\frac{1}{2})^x$ (shown in pink) and thus a reflection of $y=(\frac{1}{2})^x$ across the identity line, $y=x$ (in blue)

then "fold up" the negative part over the x-axis to account for the effect of the absolute value