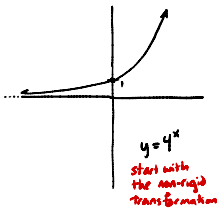
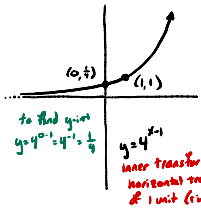
① $f(x) = |4^{x-1} + 3|$



$y = 4^x$

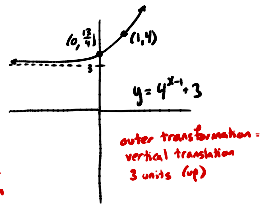
start with the non-rigid transformation



to find y-int: $y = 4^{0-1} = 4^{-1} = \frac{1}{4}$

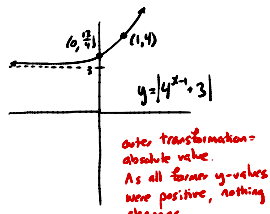
$y = 4^{x-1}$

inner transformation = horizontal translation of 1 unit (right)



$y = 4^{x-1} + 3$

outer transformation = vertical translation 3 units (up)

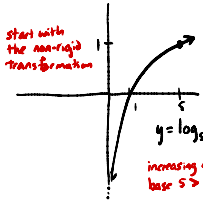


$y = |4^{x-1} + 3|$

outer transformation = absolute value. As all former y-values were positive, nothing changes.

---

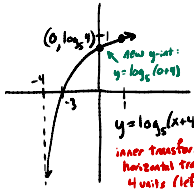
② $f(x) = \log_5\left(\frac{-|x|}{5} + 4\right)$



start with the non-rigid transformation

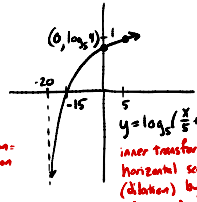
$y = \log_5 x$

increasing as base $5 > 1$



$y = \log_5(x+4)$

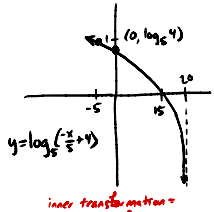
inner transformation = horizontal translation 4 units (left)



$y = \log_5\left(\frac{x}{5} + 4\right)$

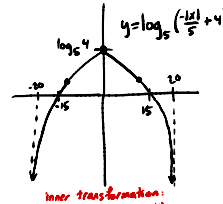
inner transformation = horizontal scaling (dilation) by a factor of 5

(it's wider now! look at the tick marks on the horiz. axis)



$y = \log_5\left(\frac{-x}{5} + 4\right)$

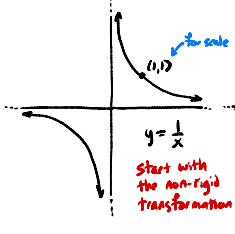
inner transformation = horizontal reflection over y-axis



inner transformation: absolute value - unlike an outer absolute value which "folds" over the x-axis, this replaces the left half with a reflection of the right half.

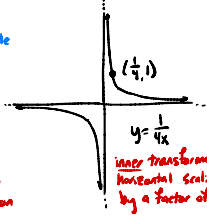
---

③ $f(x) = \frac{1}{4|x|} - 2$



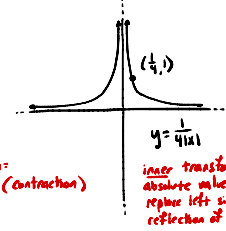
$y = \frac{1}{x}$

start with the non-rigid transformation



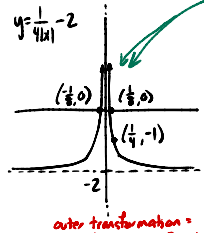
$y = \frac{1}{4x}$

inner transformation = horizontal scaling (contraction) by a factor of 4



$y = \frac{1}{4|x|}$

inner transformation: absolute value, so replace left side with reflection of right side



outer transformation = vertical translation 2 units (down)

to find x-intercepts: Solve $\frac{1}{4|x|} - 2 = 0$

$\frac{1}{4|x|} = 2$

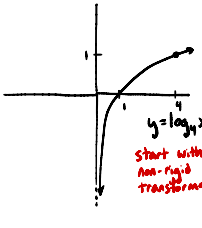
$4|x| = \frac{1}{2}$

$|x| = \frac{1}{8}$

$x = \pm\frac{1}{8}$

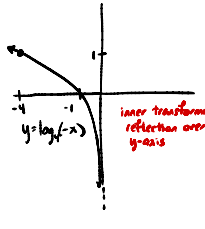
---

④ $f(x) = \left|\log_4\left(\frac{-x}{5}\right)\right|$



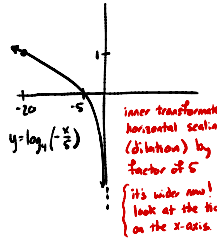
$y = \log_4 x$

start with non-rigid transformation



$y = \log_4(-x)$

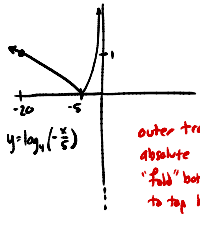
inner transformation = reflection over y-axis



$y = \log_4\left(\frac{-x}{5}\right)$

inner transformation = horizontal scaling (dilation) by a factor of 5

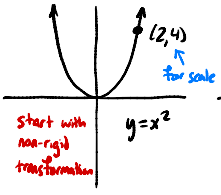
(it's wider now! look at the tickmarks on the x-axis)

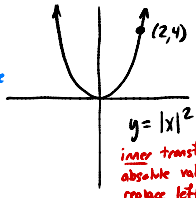


$y = \log_4\left(\frac{-x}{5}\right)$

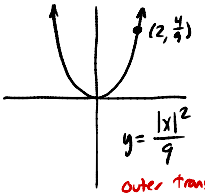
outer transformation = absolute value, so "fold" bottom half up to top half.

⑤ $f(x) = -\frac{|x|^2}{9}$

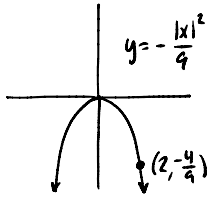




inner transformation = absolute value, so replace left half with a reflection of the right half. Note, in this case, that has no impact!



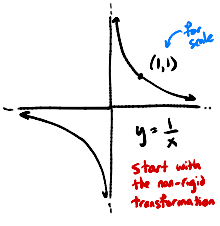
Outer transformation = vertical scaling (contraction) by a factor of 9 so points get closer to the x-axis. Look at the y-coordinate on our point previously added for scale.



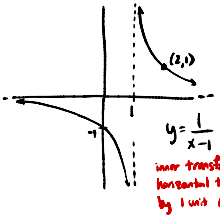
outer transformation = vertical reflection over x-axis

---

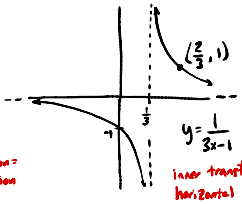
⑥ $f(x) = \frac{1}{3|x|-1}$



Start with the non-rigid transformation

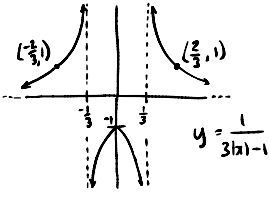


inner transformation = horizontal translation by 1 unit (right)



inner transformation = horizontal scaling (contraction) by a factor of 3

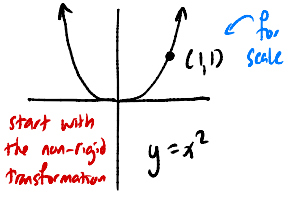
{ It got more narrow! Look at the x-coordinates associated with the vertical asymptote and the point previously added for scale }

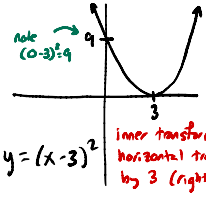


inner transformation = absolute value, so replace left half with a reflection of the right half.

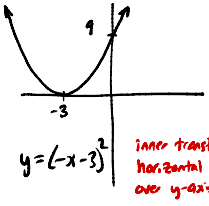
---

⑦ $f(x) = \left(-\frac{x}{4} - 3\right)^2$

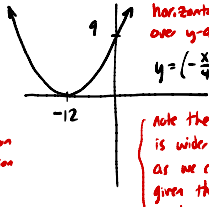




inner transformation - horizontal translation by 3 (right)



inner transformation horizontal reflection over y-axis

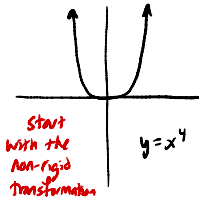


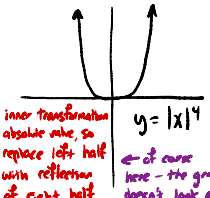
inner transformation horizontal reflection over y-axis

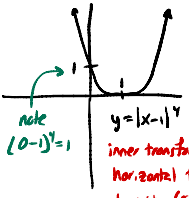
$y = \left(-\frac{x}{4} - 3\right)^2$

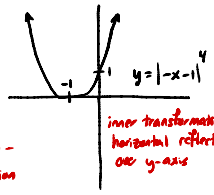
{ note the graph is wider now! as we can see given the tick mark on the x-axis }

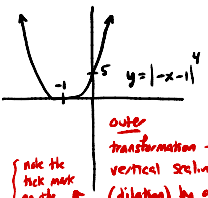
(8) $f(x) = 5|-x-1|^4$



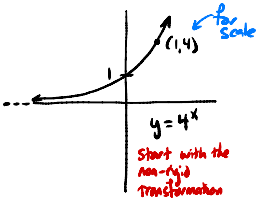


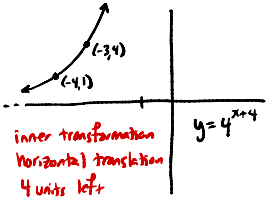


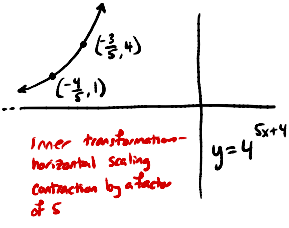


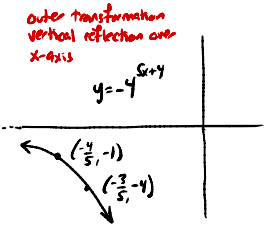


(9) $f(x) = -4^{5x+4}$

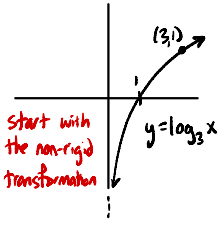


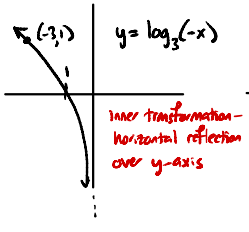


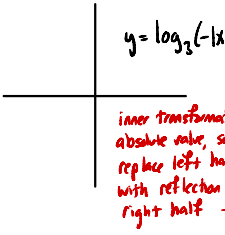




(10) $f(x) = 4\left(\log_3(-|x|)+4\right)$





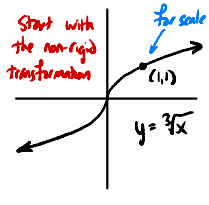


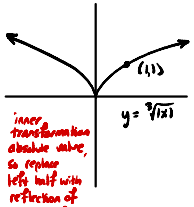
in addition to our standard approach (in red), we might also note that $-|x|$ is always negative or zero, which means the $\log(-|x|)$ will always be undefined.

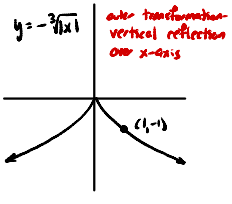
But note, as the right half was empty this leaves both halves empty

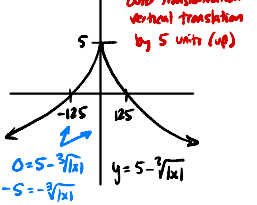
the remaining outer transformations involve shifting and then scaling an empty graph – so our final graph is empty as well! ★

(11) $f(x) = 5 - \sqrt[3]{|x|}$









$0 = 5 - \sqrt[3]{|x|}$

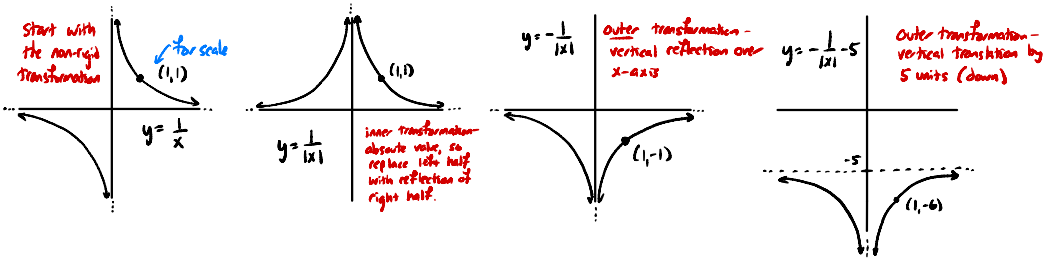
$-5 = -\sqrt[3]{|x|}$

$5 = \sqrt[3]{|x|}$

$125 = |x|$

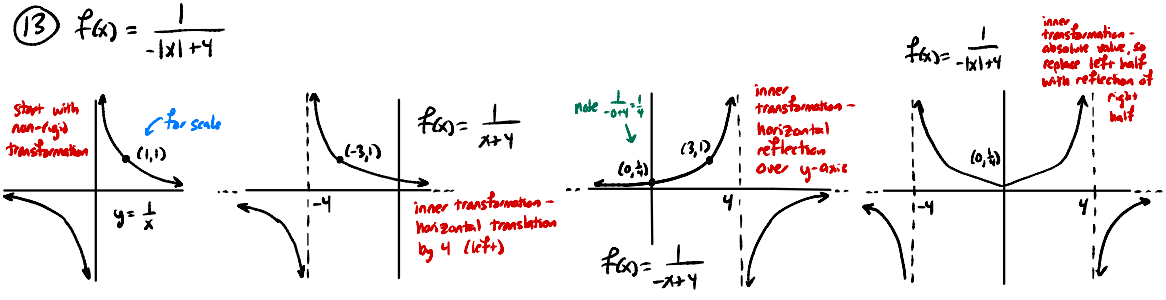
$x = \pm 125$

(12) $f(x) = -\frac{1}{|x|} - 5$



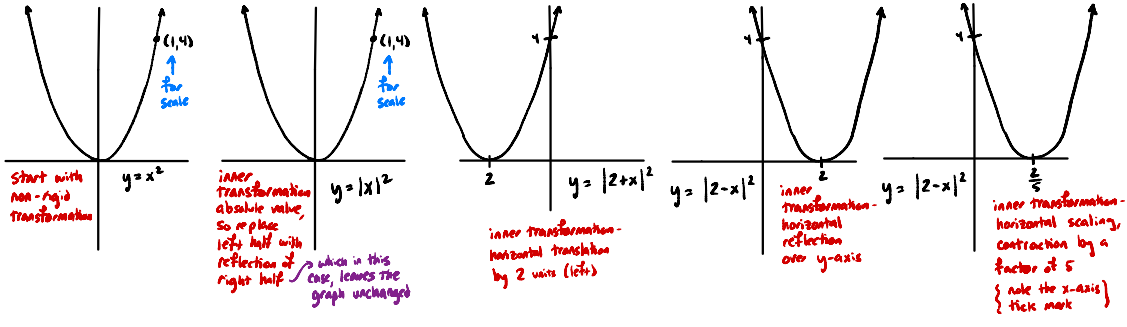
---

(13) $f(x) = \frac{1}{-|x|+4}$



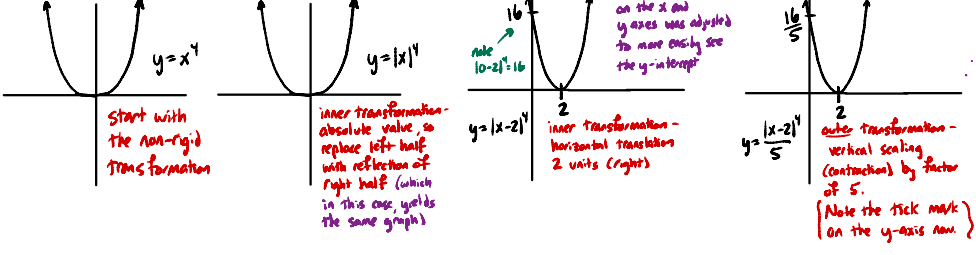
---

(14) $f(x) = |2-5x|^2$

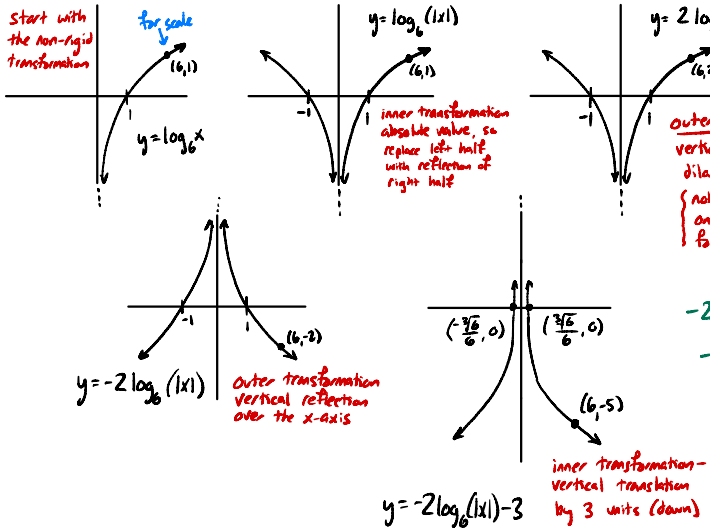


---

(15) $f(x) = \frac{|x-2|^4}{5}$



(16) $f(x) = -2\log_6(|x|) - 3$



$-2\log_6(|x|) - 3 = 0$

$-2\log_6(|x|) = 3$

$\log_6(|x|) = \frac{3}{-2}$

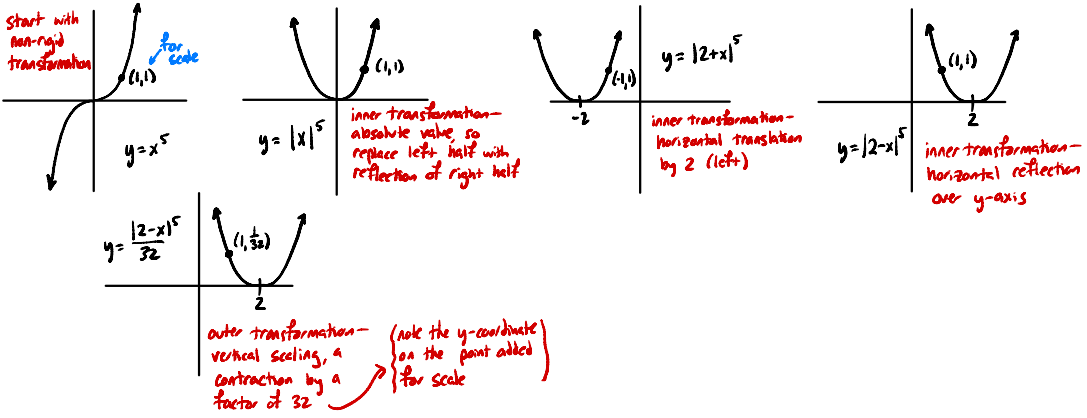
$|x| = 6^{-3/2}$

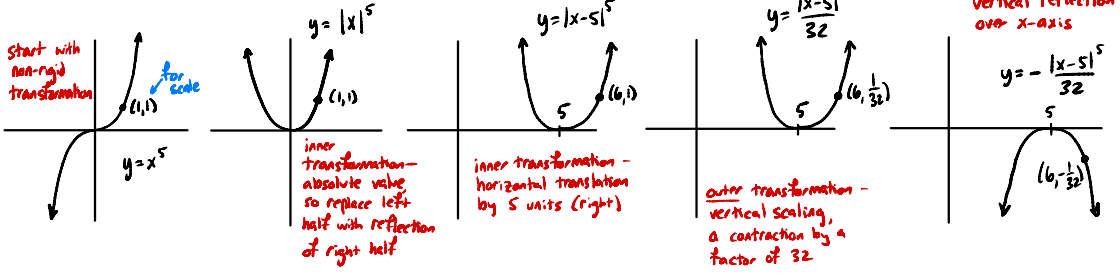
$= \frac{1}{\sqrt[3]{6^2}} \cdot \frac{\sqrt[3]{6}}{\sqrt[3]{6}} = \frac{\sqrt[3]{6}}{6}$

$x = \pm\frac{\sqrt[3]{6}}{6}$

---

(17) $f(x) = \frac{|2-x|^5}{32}$



---

(18) $f(x) = -\frac{|x-5|^5}{32}$