(1a) $2x^{-2}+3x^{-1}+1=0$

Let $u=x^{-1}$

$2u^2+3u+1=0$

$(2u+1)(u+1)=0$

$2u+1=0$ OR $u+1=0$

$2u=-1$ | $u=-1$

$u=-\frac{1}{2}$

$x^{-1}=-\frac{1}{2}$ $\quad x^{-1}=-1$

$\boxed{x=-2 \text{ OR } x=-1}$ ✓

We need to be aware that the implicit domain of what we started with on the left (i.e. $2x^{-2}+3x^{-1}+1$) doesn't include zero. So if we ended up with $x=0$, we'd have to throw that particular solution out. Luckily, this doesn't happen here — but we still need to check!

(1b) $3\left(\frac{5}{x-1}\right)^2+\left(\frac{5}{x-1}\right)-2=0$

Let $u=\frac{5}{x-1}$

$3u^2+u-2=0$

$(3u-2)(u+1)=0$

$3u-2=0$ OR $u+1=0$

$3u=2$ $\quad u=-1$

$u=\frac{2}{3}$

$\frac{5}{x-1}=\frac{2}{3}$ $\quad \frac{5}{x-1}=-1$

$\frac{x-1}{5}=\frac{3}{2}$ $\quad \frac{x-1}{5}=-1$

$x-1=\frac{15}{2}$ $\quad x-1=-5$

$x=\frac{17}{2}$ $\quad x=-4$

$\boxed{x=\frac{17}{2} \text{ OR } x=-4}$ ✓

Note, we must be alert to the fact that if we end up with a solution of $x=1$, its going to be a domain issue an will need to be thrown out. Fortunately, that doesn't happen here.

(1c) $\frac{2x-1}{x^2+2x-8}-\frac{3}{2-x}=\frac{2}{x+4}$

$\frac{2x-1}{(x+4)(x-2)}-\frac{3}{2-x}=\frac{2}{x+4}$

$\frac{2x-1}{(x+4)(x-2)}+\frac{3}{(x-2)}-\frac{2}{(x+4)}=0$

These 3 equations are equivalent, so we can use the domain restrictions on the third equation in checking our solutions as they are perhaps more obvious there (i.e., we need $x\neq 2$ and $x\neq -4$)

$\frac{2x-1}{(x+4)(x-2)}+\frac{3(x+4)}{(x+4)(x-2)}-\frac{2(x-2)}{(x+4)(x-2)}=0$

$\frac{(2x-1)+3(x+4)-2(x-2)}{(x+4)(x-2)}=0$

$\frac{\cancel{2x}-1+3x+12-\cancel{2x}+4}{(x+4)(x-2)}=0$

$\frac{3x+15}{(x+4)(x-2)}=0$

$\frac{3(x+5)}{(x+4)(x-2)}=0$

$3(x+5)=0$

$x+5=0$

$\boxed{x=-5}$ ✓

note $x=-5$ satisfies these here, so we don't have to throw it out.

(1d) $\log_{10}(2x+50)=2$

be aware, we need $2x+50>0$ so its in the domain of the log function! we could find where this is now

with $2x+50=0$

$2x=-50$

$x=-25$

and sign analysis of $(2x+50)$: $- - - -$ | $+ + + + +$ at -25

or wait and hope we see the value of $2x+50$ in our future calculations (which we do almost immediately!)

$2x+50=100$

so we are ok! ☺

$2x=50$

$x=25$ ✓

(1e) $\log_2 x + \log_2 (x-2) = 3$

$\log_2[x(x-2)] = 3$

$x(x-2) = 2^3$

$x^2 - 2x = 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$\boxed{x=4}$ ✓ or $x = -2$ ⊘

Note whatever answer we get must keep these two expressions strictly positive. That is, we require

$x>0$ and $x-2>0$

so we don't have a domain issue.

$x=-2$ doesn't satisfy the above requirement, so it is an extraneous solution which we must thus exclude!

(1f) $1 + \sqrt{2x+1} = x$

isolate the radical on one side first, as otherwise squaring both sides won't eliminate its presence.

$\sqrt{2x+1} = x-1$

$2x+1 = (x-1)^2$

Alert! We just committed an irreversible act. As such the solutions to the equations from this point forward may include values of x that are <u>not</u> solutions to the original equation. We'll need to check our final answers now and throw away any that don't work!

get everything to one side so we can try the zero-product property upon factoring

$2x+1 = x^2 - 2x + 1$

$0 = x^2 - 4x$

$0 = x(x-4)$

$x = 0$ or $x - 4 = 0$

$x = 4$

Check:

$x=0$: $1 + \sqrt{2\cdot 0+1} \neq 0$ ⊘

$x=4$: $1 + \sqrt{2\cdot 4+1} = 4$ ✓ → $\boxed{x=4 \text{ only}}$

(1g) $\log_3(7-x) = \log_3(1-x) + 1$

$\log_3(7-x) - \log_3(1-x) = 1$

$\log_3\left(\frac{7-x}{1-x}\right) = 1$

$\frac{7-x}{1-x} = 3$

We require $7-x>0$ and $1-x>0$ to avoid domain issues

$7-x = 3(1-x)$

$7-x = 3-3x$

$2x = -4$

$\boxed{x=-2}$ ✓

Note $7-(-2) = 9 > 0$ and $1-(-2) = 3 > 0$ so this really is a solution.

(1h) $\sqrt{x+1} - 3x = 1$

$\sqrt{x+1} = 1+3x$

$x+1 = (1+3x)^2$

this step is not reversible, so we'll need to check our answers...

$x+1 = 1+6x+9x^2$

$0 = 9x^2+5x$

$0 = x(9x+5)$

$x=0$ OR $9x+5=0$

$9x=-5$

$x = -\frac{5}{9}$

Check

$\sqrt{0+1} - 3\cdot 0 = 1$ ✓

Check

$\sqrt{-\frac{5}{9}+1} - 3\cdot\left(-\frac{5}{9}\right) = \frac{2}{3}+\frac{5}{3}$

$= \frac{7}{3} \neq 1$ ⊘

So $\boxed{x=0}$

(1i) $3x(x-1)^{1/2} + 2(x-1)^{3/2} = 0$

Both of these require $x-1 \geq 0$ (presuming we want solutions in $\mathbb{R}$). otherwise we have a domain issues

$(x-1)^{1/2}(3x+2(x-1)) = 0$

$(x-1)^{1/2}(3x+2x-2) = 0$

$(x-1)^{1/2}(5x-2) = 0$

$(x-1)^{1/2} = 0$ OR $5x-2=0$

$(x-1) = 0$

Not a reversible step so check solution!

$5x = 2$

$x=1$

$x = \frac{2}{5}$ ⊘

Note $\frac{2}{5}-1<0$ so we must throw out this extraneous solution!

Check!

$3\cdot 1(1-1)^{1/2} + 2(1-1)^{3/2} = 3\cdot 0 + 2\cdot 0 = 0$

So $\boxed{x=1}$ ✓

(1j) $\dfrac{\log_3 16}{2\log_3 x} = 2$

We require $x>0$ to avoid a domain issue!

$\frac{1}{2}\cdot\log_x 16 = 2$

$\log_x 16 = 4$

$x^4 = 16$

$x = \pm\sqrt[4]{16}$

$x = 2$ or $x=-2$

✓ ⊘

So $x=-2$ won't work..

(1k) $4x^{-2} - x^{-1} - 5 = 0$

Note, $x^{-1} = \frac{1}{x}$ so we require $x \neq 0$ to avoid any domain issues

Let $u = x^{-1}$

$4u^2 - u - 5 = 0$

$(4u-5)(u+1) = 0$

$4u-5=0$ OR $u+1=0$

$4u=5$

$u=-1$

$u=\frac{5}{4}$

$x^{-1}=-1$

$x^{-1}=\frac{5}{4}$

$x=-1$ ✓

$x=\frac{4}{5}$ ✓

Both are non-zero, so both avoid domain issues and are solutions!

$\boxed{x=\frac{4}{5} \text{ or } x=-1}$

(1l)

$\left(\frac{x+1}{x}\right)^2 + 2\left(\frac{x+1}{x}\right) - 3 = 0$

both of these require $x\neq 0$ to avoid domain issues.

Let $u = \frac{x+1}{x}$

$u^2+2u-3=0$

$(u+3)(u-1)=0$

$u+3=0$ OR $u-1=0$

$u=-3$ $u=1$

Both are $\neq 0$ and hence avoid domain issues and are thus solutions

(1m) $\sqrt{x} - \sqrt{x-5} = 1$

$-\sqrt{x-5} = 1-\sqrt{x}$

squaring both sides is not a reversible action so we'll need to check our solutions!

$x-5 = 1-2\sqrt{x}+x$

$-6 = -2\sqrt{x}$

$3 = \sqrt{x}$

$9 = x$

Check!

$\sqrt{9} - \sqrt{9-5} = 3-2 = 1$ ✓ it checks out! ☺

So $\boxed{x=9}$